Stamford Park Primary School

Calculation Policy

(Aligned with the National Curriculum for Mathematics)

Review Date:	Spring 2026
Review Schedule:	Annually
Person(s) Responsible:	Mr M Wharton

Introduction:

This Calculation Policy has been written in line with the programmes of study from the revised National Curriculum, (2014). It provides guidance on written calculation methods and progression for the four operations of addition, subtraction, multiplication and division. The policy is set out under these four operations, showing suggested lines of progression from the Early Years Foundation Stage through to Year 6. It also identifies the prerequisite skills and knowledge of the number system required by children before they begin to calculate.

Statements from the programmes of study are presented in bold at the start of each section for reference and recommended resources, models and images are identified throughout. This policy outlines expectations for children's counting skills and ability to read, write and order numbers where relevant. It also includes guidance relating to developing children's understanding of place value.

Children will be encouraged to use mental strategies as their first port of call where appropriate, but for calculations that they are unable to do in their heads, they will be taught to use an efficient written method.

Aims:

- To ensure consistency and progression in our approach to calculation from Nursery through to Year 6
- To ensure children develop efficient, reliable, formal written methods of calculation across all four operations
- To ensure children develop a sound understanding of place value to support them with their calculations
- To ensure that children develop rapid mental recall of key number facts which are appropriate to their stage of learning and, where appropriate, are able to effectively apply mental strategies to support their calculations
- To ensure that children can use all methods taught accurately, with confidence and understanding.

Key Points for Staff:

- This policy should be used as the basis for planning for calculation whilst making reference to the previous/ following year group to ensure personalised learning. If, at any time, children are making continuous errors, return to the previous stage of calculation
- It is important to note that this is a calculation policy and does not necessarily cover the use and application of calculation strategies.
- **There is no requirement for children to use formal written methods for recording their calculations in the EYFS.**
- The new curriculum in particular places great store in being able to select and use the appropriate method for a given problem and should be given plenty of opportunities to use and apply their knowledge in a range of situations and contexts including problems requiring single and multiple step calculations, trial and error and open ended investigations. Always encourage the children to make appropriate choices about which methods to use when solving problems and use diagrams, tables and jottings to help them to find a way to break down a problem.

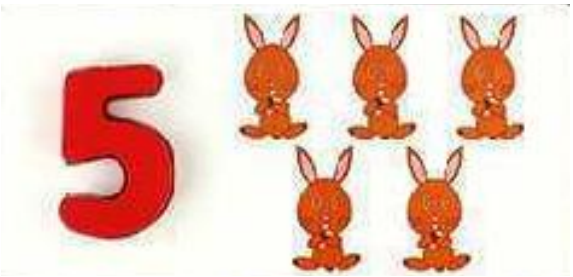
SECTION ONE

Calculation in Early Years Foundation Stage and Key Stage One

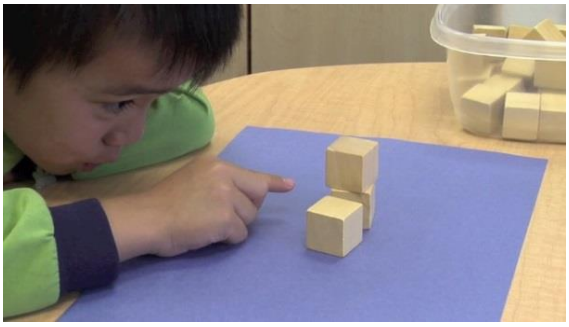
Calculation in Early Years Foundation Stage and Key Stage One - Prerequisite Skills

The following knowledge and skills should be in place before moving on to carrying out calculations:

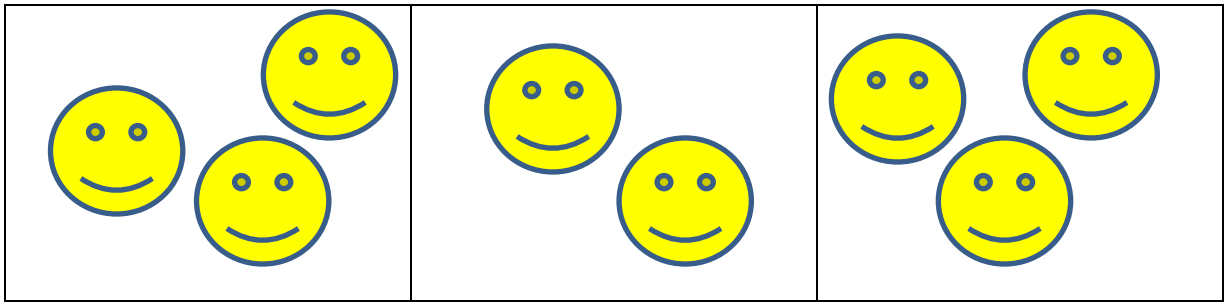
- Recite number names in order – up to at least 5, and then up to 10
- Count up to at least five objects accurately using one to one correspondence, (saying one number name for each object)
- Cardinality - know that the last number said when counting represents the total in the set
- Know that numbers identify how many objects are in a set and beginning to recognise some written numerals.



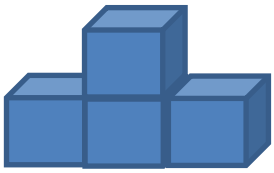
- Accurately count a small group of objects that can't be moved
- Accurately count out a small group of objects from a larger set

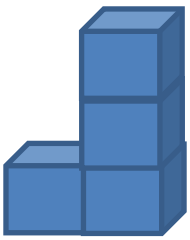


- Subitising – recognise groups of one, two or three objects without counting



- Conservation of number – move around, partition or recombine small groups of objects, (up to 4 or 5), and recognise that the total remains unchanged.





- Know that zero represents an empty set

(For further guidance on how to support children in developing these skills refer to Numbers and Patterns: Laying Foundations in Mathematics, 2009)

Stages in Addition

In developing a written method for addition, it is important that children understand the concept of addition, in that it is:

- Combining two or more groups to give a total or sum
- Increasing an amount

In order to carry out their calculations relating to addition efficiently, they also need to develop an understanding that it is:

- The inverse of subtraction
- Commutative i.e. 5 + 3 = 3 + 5
- Associative i.e. 5 + 3 + 7 = 5 + (3 + 7)

Addition – Early Years Foundation Stage

ELGs

Number:

Pupils should be taught to:

*Have a deep understanding of number to 10, including the composition of each number.

*Subitise (recognise quantities without counting) up to 5.

*Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to ten, including double facts.

Numerical Patterns:

Pupils should be taught to:

*Verbally count beyond 20, recognising the pattern of the counting system.

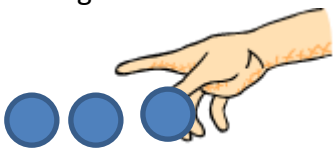
*Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as another quantity.

*Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed evenly.

Vocabulary:

In practical activities and through discussion they will begin to use the vocabulary involved in addition – add, more, plus, total, altogether, the same as, makes.

Children will begin to add two sets of objects by putting them together and counting them all. They should be encouraged to move each object as they count/ arrange them in a line to avoid miscounting.



Children practice counting on from a given number - within ten and then 20. This helps them as they then progress to **counting on** from the first set. They will respond to simple questions involving addition such as:

You have 5 biscuits and I have 2. How many biscuits altogether?

"five……..six….seven"

To encourage this, once the quantity in the first set has been established, this set can be covered/ labelled with the corresponding numeral.

"five…….six….seven"

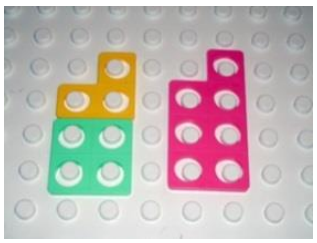
Numicon is also used to support addition. The children become familiar with the pieces and the quantity each represents by using them in their play. For example, they use them as printing blocks in the painting area, make marks with them in the Playdoh and use them to organise small items of loose parts such as beads or stones. They then use them to add two numbers by selecting the pieces that correspond to each number and fitting them together.

As with the example above, they will first add them by counting all the holes, then by counting on – see below.

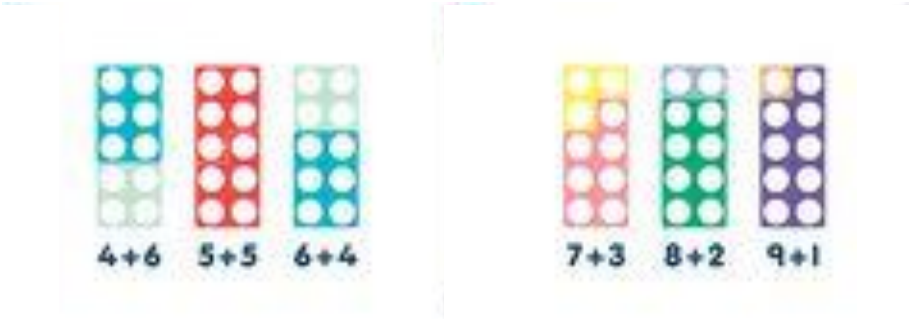
"One, two, three, four, five, six, seven"

“Four, five, six, seven”

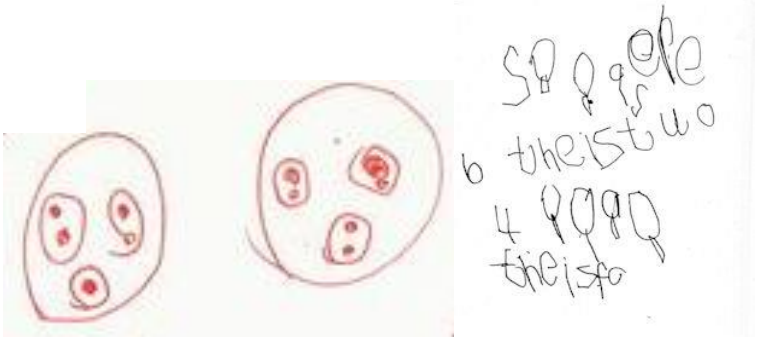
The children are then encouraged to match this to the piece of Numicon that represents 7, by placing it next to the two pieces they have placed and then showing that it matches them by placing it on top. As the children internalise the shape of each piece they will start to recognise this as seven without the need for counting.

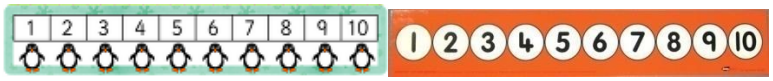


By using the Numicon in this way, the children will also start to explore number bonds to make totals up to 10 and begin to develop mental recall of these.



The children will move on from using real objects and counters to using their fingers/ drawing pictures or marks to represent them.



	Number tracks are introduced indoors and outdoors to support number recognition and counting on, starting with tracks with numbers **and** objects before moving onto just numbers.  Counting on using a number line is also modelled by adults using a variety of board games.

Addition – Year One

Related NC Statutory requirements for Year 1:

Pupils should be taught to:

*Given a number, identify one more.

*Read, write and interpret mathematical statements involving addition (+) and the equals sign (=)

*Represent and use number bonds within 20

*Add one-digit numbers and two-digit numbers within 20, including zero

*Solve one step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as

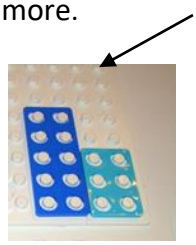
13 = ? + 6

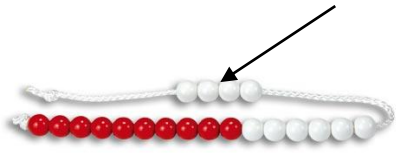
*They will also carry out practical activities which involve ordinal numbers – first, second, third etc.

Vocabulary:

Children's mathematical vocabulary continues to be built upon. Problems should include the terms; put together, add, more, more than, plus, altogether, total, equals, equal to, double, most, count up, count on, difference between, distance between.

Children use Numicon and bead strings to make numbers up to 20 which helps them to begin to develop an understanding of the place value of a two-digit number, for example by seeing 16 as 10 and 6 more.

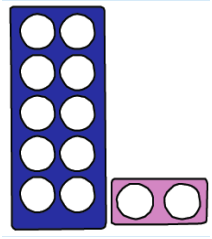




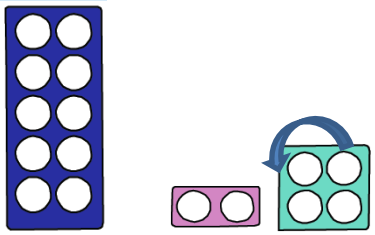
Children consolidate rapid recall of number bonds for numbers up to ten, including recall of doubles. They apply their knowledge of pairs of numbers that make ten to help them to identify pairs of numbers to make twenty.

9 + 1 = 10 19 + 1 = 20

Children will initially continue to use objects to carry out their calculations. They use Numicon and Dienes, (Base ten), apparatus to make teens numbers… and then to add on a single digit number.

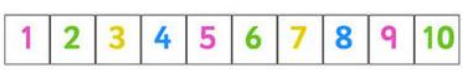


12



12 + 4 = 16

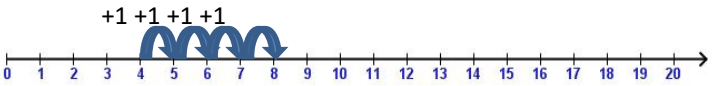
Children then progress to using a number track, (or number ladder), to count on for addition. They are taught to count on from the largest number.



6 + 2 = 8

"Put your finger on the number 6. Count on two more."

Children then progress to a marked number line:



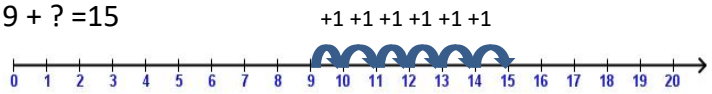
4 + 5 = 9

"Put your finger on the 7 and count on four."

Children use objects and then number tracks/ number lines to help them to solve addition problems involving missing numbers:

11 + 3 = ? ? + ? = 8

9 + ? =15



9 + 6 = 15

Ensure children are confident with using a marked number line before moving on to an empty number line, (see Year Two guidance).

Children continue to practice counting on from a given number between zero and 20 and then beyond. Those who are confident in adding two numbers by counting on from the first may use their fingers to do so. They can then count in this way to solve addition problems for missing numbers.

7 + ☐ =11

"Seven…eight, nine, ten, eleven"

7 + 4 = 11

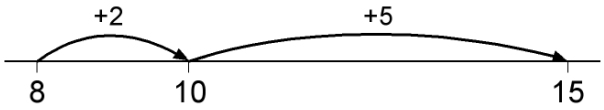
Bead strings or bead bars are used to illustrate addition, in particular for demonstrating bridging through ten.

7 + 6 = 13

As their rapid recall of number bonds increases children are encouraged to use this knowledge to help them when bridging through 10.

E.g.

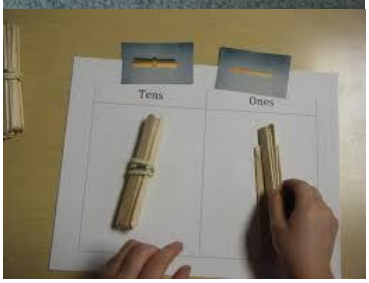
8 + 7 = 8 + 2 + 5 = 15



To begin to develop their understanding of place value, children begin to count out straws or lolly sticks and create bundles of ten, (see below), where the number of bundles represents the number of tens and the loose straws represent the number of ones. This helps children to appreciate place value and understand which digit will change if they increase a number by one or ten.

Here children have used straws to represent the number of children in the class.
10 + 4 = 14

Here the child has used straws to show 33.
10 + 10 + 10 = 30
1 + 1 + 1 = 3
30 + 3 = 33



Here the child has made 17 with lolly sticks.
10 + 7 = 17

Addition – Year Two

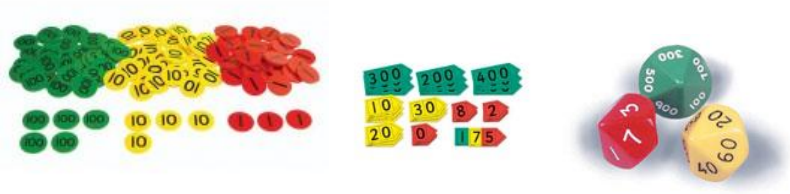
Related NC Statutory Requirements for Year 2:
Pupils should be taught to:
*Add numbers using concrete objects, pictorial representations, and mentally, including:
*A two-digit number and ones
*A two-digit number and tens
*Two two-digit numbers
*Three one-digit numbers

Vocabulary:
Add, more, plus, and, make, altogether, total, equals, equal to, double, most, count on, sum, tens, ones, partition, addition, column.

Ensure that children are confident with the methods outlined in the Year One guidance before moving on.

In addition to the bundles of ten introduced in Year One to Dienes, (Base ten), are used to further support children's developing understanding of place value. These help to illustrate the value of each digit. Although mainly working with two digit numbers in their calculations, children are encouraged to use these resources to represent three digit numbers too.

Over time these resources may be replaced with place value counters, (see below).



(The counters are colour coded to match place value arrows and dice.)

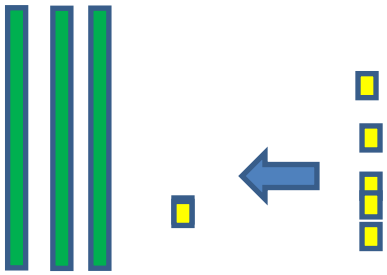
Grids with separate columns to represent each digit are also used for support.

Hundreds	Tens	Units

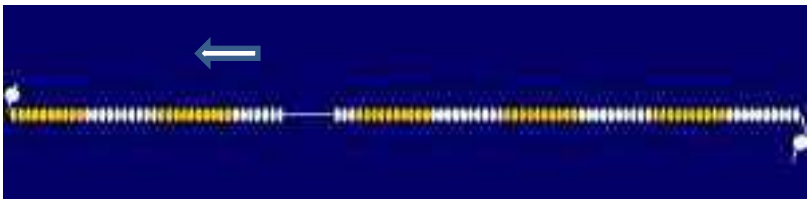
Adding a two digit number and ones/ counting on in ones:

32 + 5 = 37

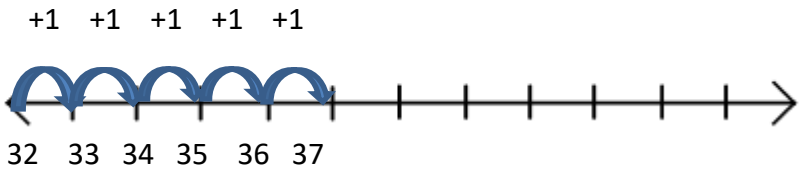
Using Dienes:



Using bead strings:



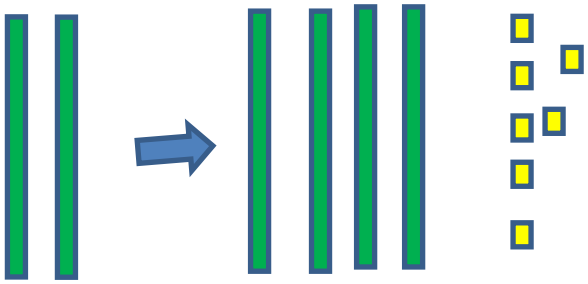
Using a number square or an empty number line, within 100:



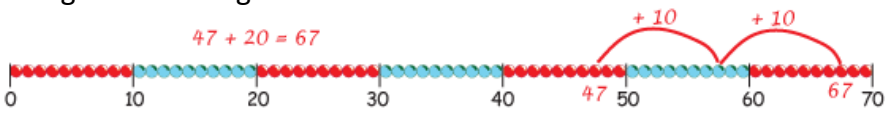
Adding a two digit number and tens/counting on in tens:

47 + 20 = 67

Using Dienes:

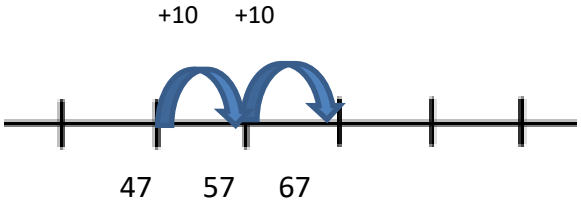


Using a bead string:



Using an empty number line:

47 + 20 = 67



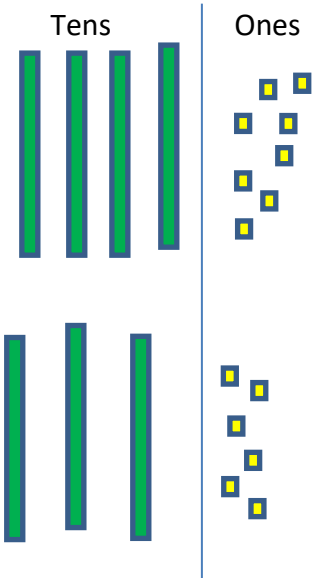
100 squares are used in conjunction with the other resources mentioned, in particular to show jumps of ten.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

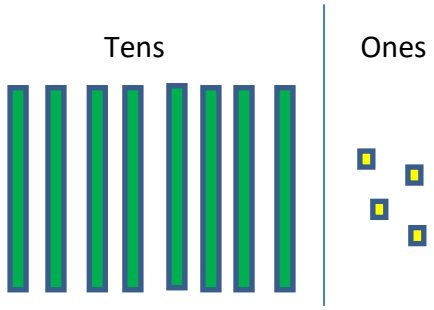
Adding pairs of two digit numbers:

In the example below Dienes are used. The use of these resources helps to demonstrate the partitioning method.

Using Dienes: 48 + 36

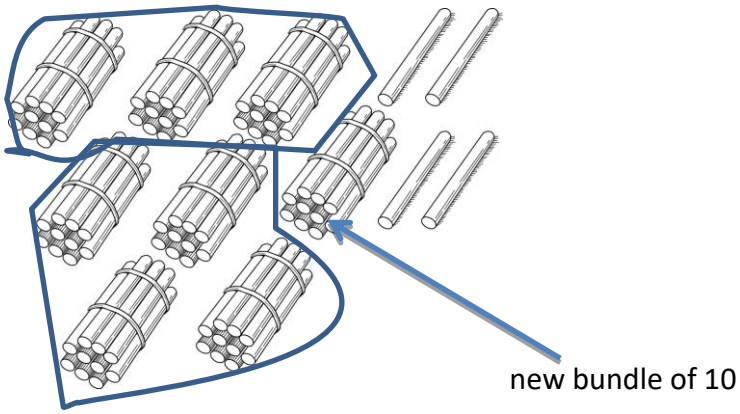


Ten of the ones are exchanged for for a ten stick to find the total of 84



80 + 4 = 84

Sticks/ straws are also useful for this as children can make another bundle of ten by fastening ten loose sticks/ straws together to make another bundle.



48 + 36 = 84

In this example we start with 4 bundles of 10, (40), and 8 loose rods, and 3 bundles of 10, (30), and 6 loose rods (36). This gives us 7 bundles of 10 and 14 loose rods. Ten of the loose rods are then used to make another bundle. We now have 8 bundles of 10, (80), and 4 loose rods, making 84.

Children can then use this method without the need for apparatus:

Addition of two 2 digit numbers using the partitioning method: (without bridging through ten)

43 + 25 = 68

40 3 20 5

(tens) 40 + 20 = 60
(ones) 3 + 5 = 8
60 + 8 = 68

(bridging through ten)

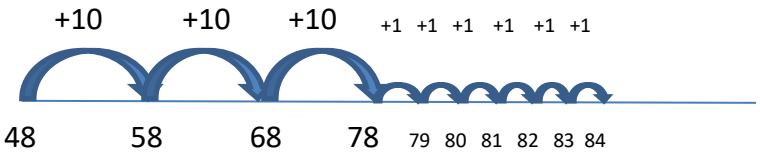
48 + 36 = 40 + 8 + 30 + 6
(tens) 40 + 30 = 70
ones) 8 + 6 = 14
70 + 14 = 70 + 10 + 4 = 80 + 4 = 84

Using a number line to count on:

48 + 36 = ?

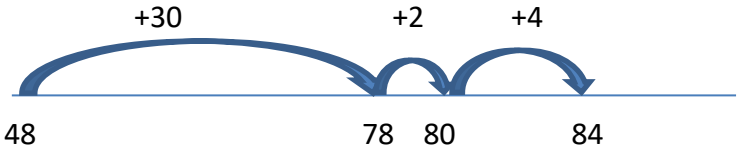
"Put the largest number first, (48), and then partitioning the smaller number, (36 = 30 + 6), and counting on –

48 + 30 + 6



Use in conjunction with a 100 square to show jumps of tens and ones.

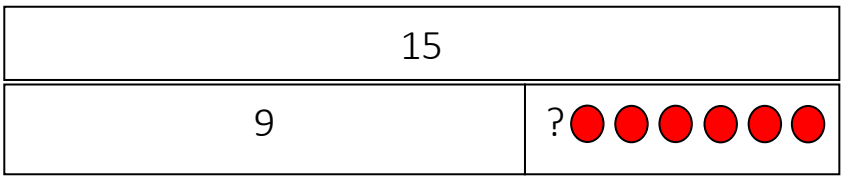
If children are confident with this, they should be encouraged to use more efficient jumps:



Use in conjunction with a 100 square as before.

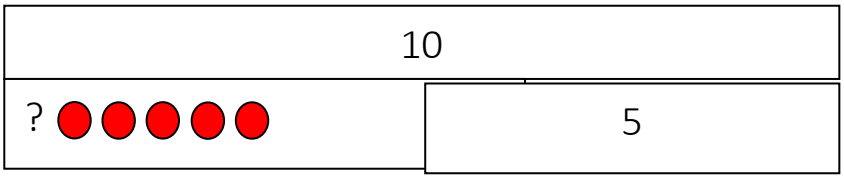
<u>Using a 'Bar Model' to find a missing number problem:</u>
Children may use a 'Bar Model' to help visualise and solve a missing number addition problem such as,

9 + ? = 15



Children can then 'count on' to find the unknown.

? + 5 = 10



Also, children can then 'count on' to find the unknown.

Stages in Subtraction

In developing a written method for subtraction, it is important that children understand the concept of subtraction, in the form of:

- Removal of an amount from a larger group (take away)
- Comparison of two amounts (difference)

They also need to understand and work with certain principles, i.e. that it is:

- The inverse of addition
- Not commutative i.e. 5 – 3 is not the same as 3 – 5
- Not associative i.e. 10 – 3 – 2 is not the same as 10 – (3 – 2)

Subtraction – Early Years Foundation Stage

ELGs

Number:

Pupils should be taught to:

*Have a deep understanding of number to 10, including the composition of each number.

*Subitise (recognise quantities without counting) up to 5.

*Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to ten, including double facts.

Numerical Patterns:

Pupils should be taught to:

*Verbally count beyond 20, recognising the pattern of the counting system.

*Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as another quantity.

*Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed evenly.

Vocabulary:

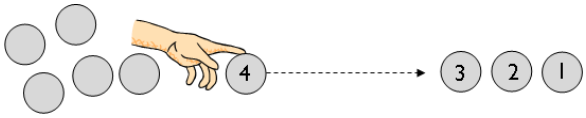
In practical activities and through discussion they will begin to use the vocabulary involved in subtraction – take away, subtract, minus, less, count back, how many, left, fewer.

Children practice counting back from a given number, first from a number between 10 and 0, then 20 and 0. Children use pictures and props to accompany songs and rhymes involving one less, counting back or taking away.

Children will begin to develop their ability to subtract by using practical equipment to count out the first number and then remove or take away the second number to find the solution by counting how many are left.

"If I have four biscuits and I eat two how many will I have left?"

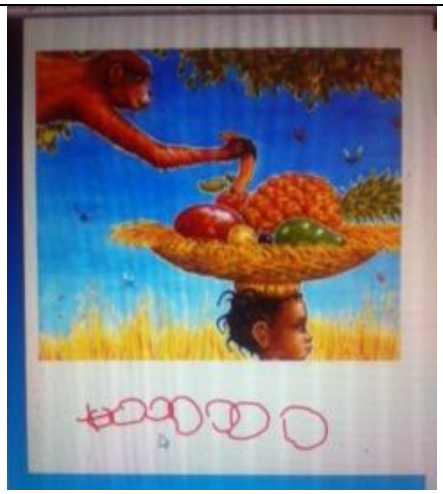
"What do I get if I take four from nine?"
"What is nine minus four?"



Children use bead strings to subtract a single digit number from a number up to ten.

"Ten take away six"

They may begin to record their calculations by drawing objects or marks and then crossing some out or rubbing them out.

Subtraction – Year One

Related NC Statutory Requirements for Year 1:

Pupils should be taught to:

*Given a number, identify one less

*Read, write and interpret mathematical statements involving subtraction (-) and the equals (=) sign

*Subtract one-digit and two-digit numbers within 20, including zero

*Solve missing number problems e.g.

20 - ☐ =15

Vocabulary:

Equal to, take, take away, less, minus, subtract, leaves, difference, distance between, how many fewer, less than, most, least, count back.

Children continue counting back from a given number within twenty.

They continue to use practical equipment to solve subtraction problems by "taking away". This is done using objects and resources such as counters and blocks or Multilink cubes. Dienes, (Base ten), cubes may be used, but not the tens sticks at this stage. This allows the children to subtract by moving the counters or cubes away.

E.g. 13 - 4

They subtract by counting back using number tracks, (or number ladders):

6 – 3 = 3

"What is three less than six?"

"Start on six and count back three"

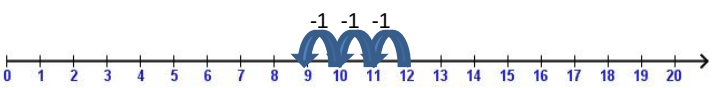
They then progress to using a marked number line:

14 – 5 = 9

NB Ensure children are confident with using a marked number line before moving on to an empty number line, (see Year Two guidance)

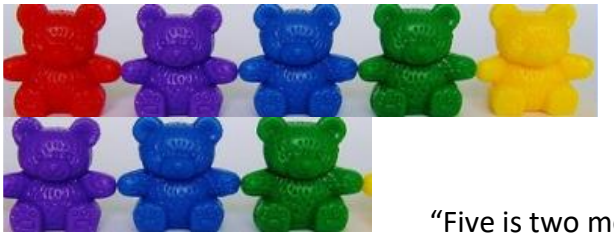
Number tracks and number lines are also used to solve subtraction problems involving missing numbers by counting back.

12 - ☐ = 9 12 – 3 = 9



Counting on to find a small difference:

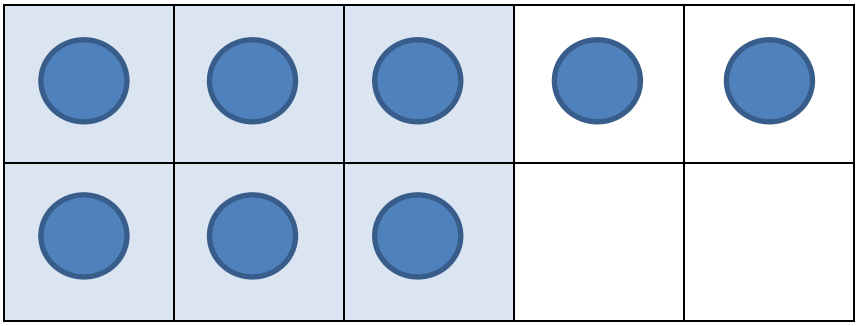
Children begin to use two sets of objects to help them to find the **difference** between two numbers by lining them up together; ensuring that each set is equally spaced to avoid any miscalculation. The use of models is extremely important here to understand the idea of "difference"



"Five is two more than three"

"What is the difference between three and one?"

Grids can help with this:

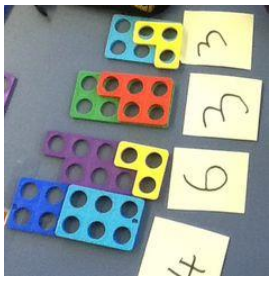


"One…….Two……..The difference is two"
5 – 3 = 2

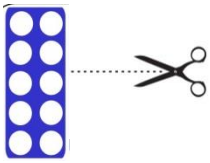
Egg boxes can also be used as a grid.

Objects which are identical tend to be more helpful for this and interlocking blocks such as multilink cubes are particularly useful as they can be placed horizontally or vertically adjacent to one-another to draw attention to the difference.

Children are taught to **count up** from the smallest number to the largest to **find the difference** using these resources.
Numicon pieces are placed on top of one another to highlight the difference.



8 – 6 = 2



Paper Numicon pieces can be used to "cut away" the smaller number*

10 – 6 = 4

"The difference between 10 and 6 is 4"

*This is also a good way of showing children how to apply their knowledge of number bonds to find missing numbers,

as in 10 - ☐ = 4 or 10 - ☐ = 6

Subtraction – Year Two

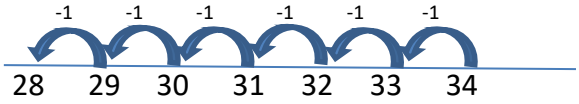
Related NC Statutory Requirements for Year 2:
Pupils should be taught to:
*Subtract numbers using concrete objects, pictorial representations, and mentally including:
*A two-digit number and ones
*A two-digit number and tens
*Two two-digit numbers

Vocabulary:
Equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is…?

Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

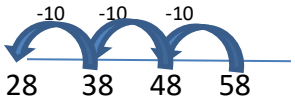
Children count back on an empty number line within 100, in ones…..

34 – 6 = 28



…and in tens:

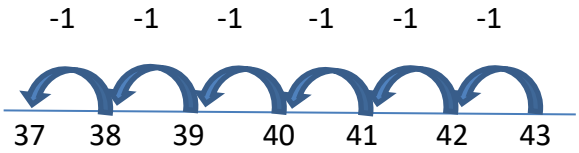
58 – 30 = 28



The number line may be used in conjunction with a 100 square to show jumps of tens.

Children also use a number line for counting back to solve subtraction problems with missing numbers:

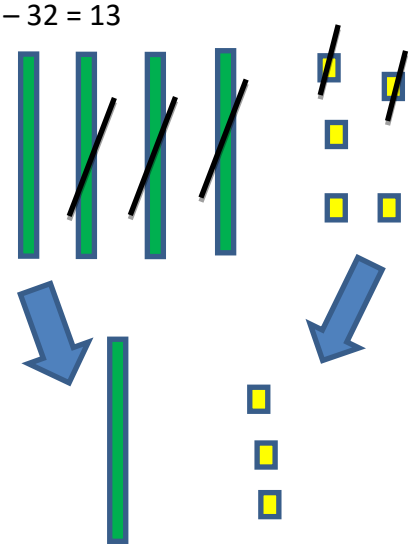
43 – □ = 37



43 – 6 = 37

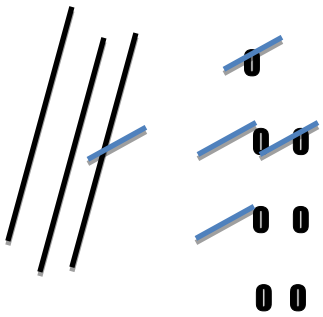
Subtraction of two two-digit numbers:
Children use the Dienes, (Base 10) equipment to support their calculations, using a take away, (or removal), method. As with addition, they need to have a secure understanding of the place value of each digit. They also need to know to start by selecting the correct number of tens and ones to make the larger number before removing the value of the smaller number to find the solution. They start with subtracting numbers where there is no exchange required, (see below).

45 – 32 = 13



Children may use their own markings rather than the Dienes apparatus to support their calculations :

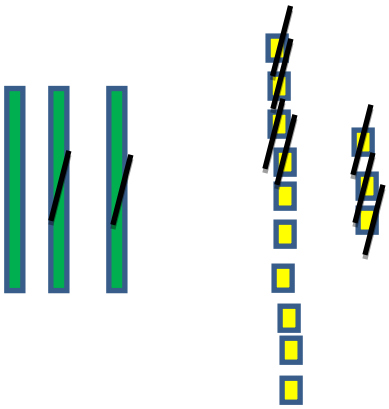
37 – 14 = 23



<u>Subtracting two two-digit numbers with an exchange:</u>

43 – 27

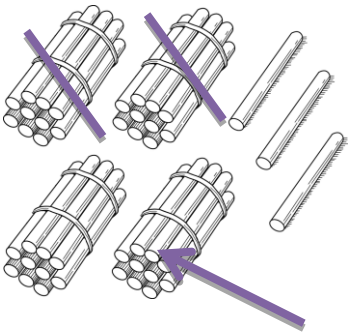
Where the number being subtracted has more ones than the larger number, one of the tens is exchanged for ten ones before taking the smaller number away.



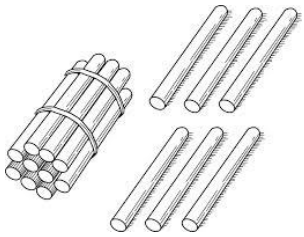
43 – 27 = 16

Some children may find this easier to understand by using bundles of straws to represent the tens as they can separate a bundle rather than needing to exchange it for something else…..43 – 27 using bundles:

Take 20 away (2 bundles of 10)



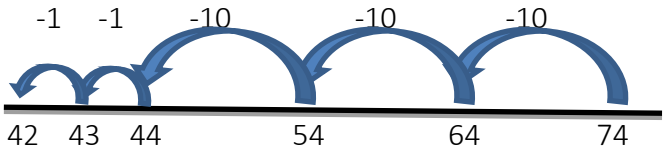
As there are not enough individual sticks to take the 7 away, one bundle is split before removing 7 sticks



This leaves one bundle (10) and 6 loose sticks(6)
Answer = 16

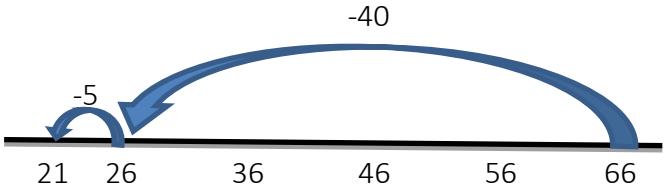
Children with a good understanding of place value and partitioning numbers may use an unmarked number line to subtract:

74 – 32 = 42



If children are confident with this, they can use more efficient jumps:

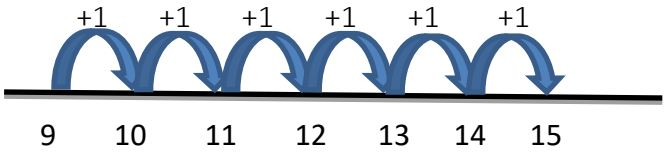
66 – 45 = 21



<u>Counting on to find a *small* difference:</u>

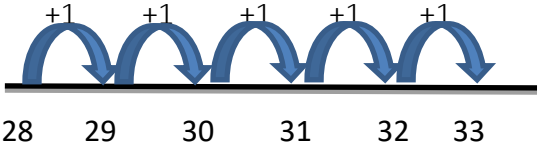
Children count up from the smallest number to the largest to **find the difference**:

15 – 9 = 6



"The difference between 9 and 15 is 6"

33 – 28 = 5

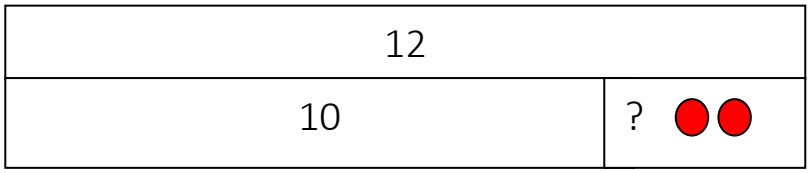


"The difference between 28 and 33 is 5"

Using a 'Bar Model' to solve a missing number subtraction problem:

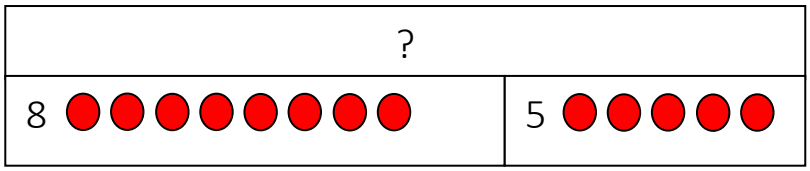
Children may use a 'Bar Model' to help visualise and solve a missing number subtraction problem such as,

12 – ? = 10



Children can then 'count on' to find the unknown.

? – 5 = 8



Children are taught to recognise that the initial number in a subtraction is the 'biggest number'. Facts such as these help to set up the bar model (see above) as the two numbers must be 'added' together to find the unknown.

8 + 5 = 13

Stages in Multiplication

In developing a written method for multiplication, it is important that children understand the concept of multiplication, in that it is:

- Repeated addition

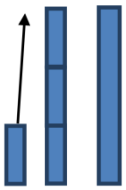
They should also be familiar with the fact that it can be represented as an array

They also need to understand and work with certain principles, i.e. that it is:

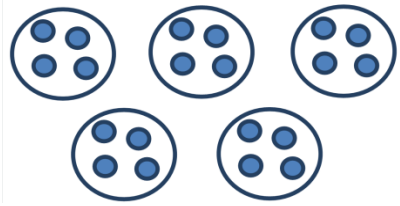
- The inverse of division
- Commutative i.e. 5 x 3 is the same as 3 x 5
- Associative i.e. 2 x 3 x 5 is the same as 2 x (3 x 5)

There are two structures for multiplication, these are **scaling** and **grouping**.

Scaling (ratio/ increasing in equal parts of something). This is first met when children explore doubling.



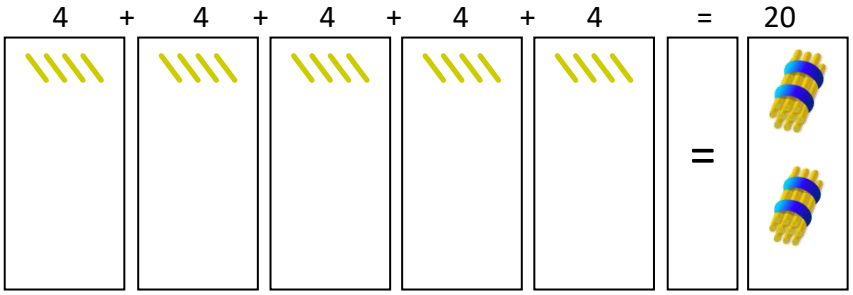
Grouping is repeated addition.



5 lots/groups of 4

4 multiplied by 5

4 x 5



4 five times

Multiplication – Early Years Foundation Stage

ELGs

Numerical Patterns:

Pupils should be taught to:

*Verbally count beyond 20, recognising the pattern of the counting system.

*Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as another quantity.

*Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed evenly.

Vocabulary:

In practical activities and through discussion they will begin to use the vocabulary involved in multiplication – groups/ groups of, lots of, sets, doubles.

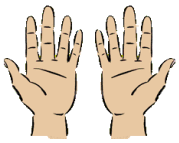
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They are given a wide experience of practical calculation opportunities using a variety of equipment, including small world play, role play, counters, cubes etc.

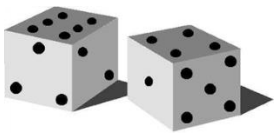
Children start to learn about doubling and think about equal groups or sets of objects in their play. They may start to represent doubles in their mark making, (see below).

Showing 5 digits on each hand

Showing 3 cookies each to help them find the total when problem solving

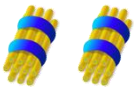
Children begin to develop mental recall of some doubles through these experiences. They quickly recognise double 5 as 10 through using their fingers when counting and doubles such as double 2 and double 3 through their use of dice when playing games.





They are introduced to counting orally in multiples of ten. Resources such as 10 Numicon blocks or bundles of 10 sticks/ straws are shown as they count to reinforce the unit they are counting in.

"Ten, twenty...etc."



Children may also investigate putting groups or individual items into resources such as egg boxes, ice cube trays and baking tins which are arrays.

Multiplication – Year One

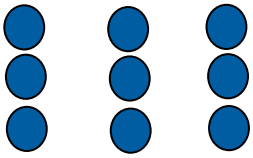
Related NC Statutory Requirements for Year 1:
Pupils should be taught to: Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Vocabulary:
Groups/ groups of, lots of, sets, doubles, multiple (as in multiple of 2/ 5 etc.), steps of (as in steps of 2….).

In Year One children will begin to understand multiplication by continuing to use a variety of objects to make sets and groups of objects. They solve multiplication problems using practical equipment and jottings.

'If each snowman needs 3 buttons, how many will we need for 3 snowmen?'

(Child uses counters to solve)

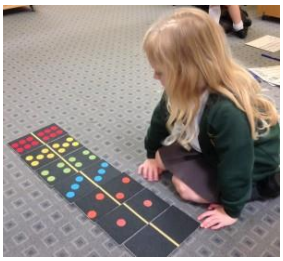


"Nine buttons"

Children are shown everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc. and use this in their learning, answering questions such as

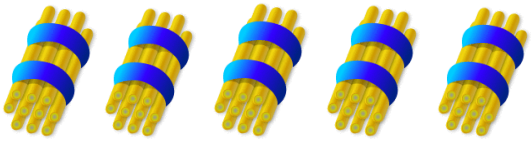
'How many eggs/ cakes would we need to fill the box? How do you know?'

They continue to use Numicon and other resources such as dominoes and dice to explore doubles and begin to extend their mental recall of these.



Children are encouraged to extend their knowledge of counting in 10's and practice counting orally in 5's and 2's, first starting from zero before counting on from different multiples of these numbers, such as "Count in fives starting at 30". They use their hands, (one at a time for 5's and both together for 10's), as they say the numbers to remind them of the unit they are counting in.

To demonstrate their growing knowledge of place value children are asked to make multiples of 10 using bundles.



"Five 10's – that makes 50"

Multiples are highlighted on a number line or hundred square as they say them to draw attention to the pattern.

For example,

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Through these activities, children are taught to make connections between arrays, number patterns and counting in twos, fives and tens.

Multiplication – Year Two

Related NC Statutory Requirements for Year 2:

Pupils should be taught to:

*Recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers

*Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×), and equals (=) signs

*Show that multiplication of two numbers can be done in any order (commutative)

*Solve problems involving multiplication using materials, arrays, repeated addition, mental methods and multiplication, including problems in contexts

Vocabulary:

Lots of, groups of, times, multiply, multiplied by multiple of once, twice, three times… ten times…times as (big, long, wide… and so on) repeated addition, array row, column, double.

Numicon is used as a visual aid for introducing odd and even numbers:

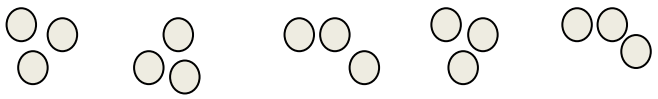
By placing the even pieces in a row the children are able to see how they are built up in two's. They can also relate this to their knowledge of counting orally in two's.

Hundred squares are used to help children to recognise the patterns made by odd and even numbers:

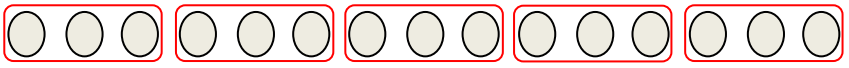
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Children are taught to understand and be able to calculate multiplication as repeated addition, supported by the use of practical apparatus such as counters or cubes.

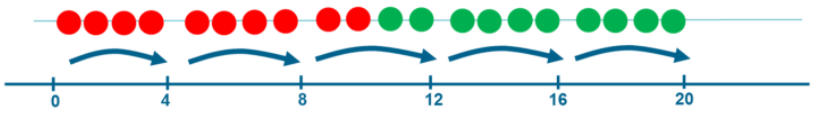
e.g. 5 x 3 can be shown as five groups of three with counters, either grouped in a random pattern, as below:



Or… in a more ordered pattern, with the groups of three indicated by the border outline:



Number lines are also used to support repeated addition.



Children develop their use of representing multiple groups with counters to show how multiplication calculations can be represented by an array, (this knowledge will support with the development of the grid method in the future).

As well as counters children are encouraged to use practical apparatus and pictures or stickers on square paper to support their understanding.
e.g.
5 x 3* can be represented as an array in two forms (as it has commutativity):

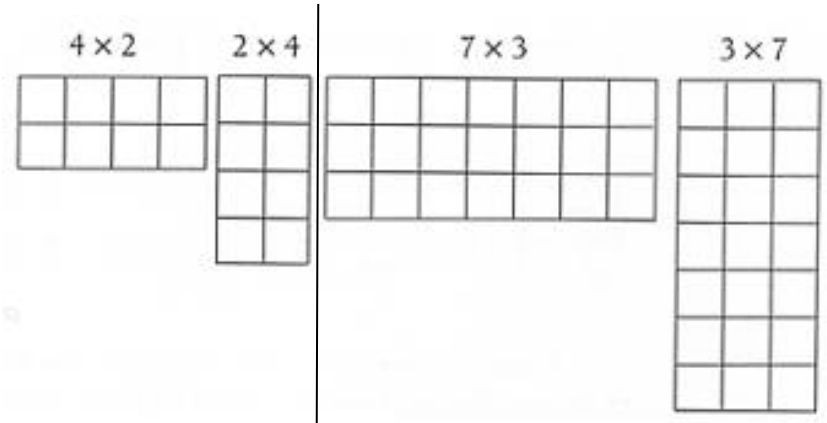
3 + 3 + 3 + 3 + 3 = 15
3 x 5 = 15

5 + 5 + 5 = 15*
5 x 3 = 15

Children initially record this as repeated addition and are then introduced to the multiplication symbol alongside this.

For mathematical accuracy 5 x 3 is represented by the second example above, rather than the first as it is five, three times. However, because we use terms such as 'groups of' or 'lots of', children are more familiar with the initial notation. (Due to the commutative order of multiplication the order is irrelevant and both ways should be taught together – as shown above – to reinforce this).

Square paper used to demonstrate arrays, (these can be rotated and placed on top of each other to demonstrate that they are the same):



4x2 = 4+4 = 8

2x4 = 4+4+4+4 = 8

4x2 = 2x4

7x3 = 7+7+7 = 21

3x7 = 3+3+3+3+3+3+3 = 21

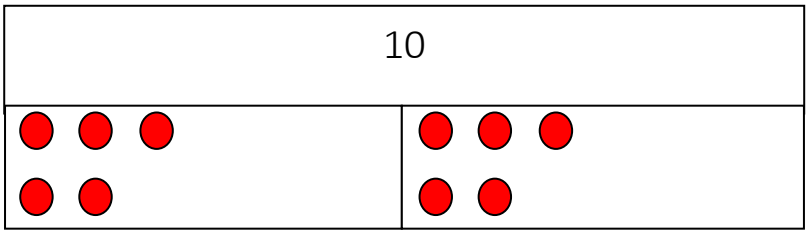
7x3 = 3x7

Children are encouraged to use the number facts they know to solve problems wherever possible. In particular, when solving problems involving doubles or multiples of 2, 5 and 10, including money problems.

<u>Using a 'Bar Model' to solve a multiplication problem:</u>

Children may be asked to solve multiplication problems such as 2 x ? = 10 .

A bar model can be used to help 'visualise' this problem and the children would set it up in the following way.



Children may then use their knowledge of doubles or number bonds to 'chunk' the ten into two groups.

Alternatively, children can 'share' the 10 equally between the 2 groups, revealing the two groups/sets of 5 which make 10.

Stages in Division

The models for division are the same as those for multiplication which means that they should be relatively easy to teach together.

In developing a written method for division, it is important that children understand the concept of division, in that it is:

- Repeated subtraction
- Sharing into equal amounts

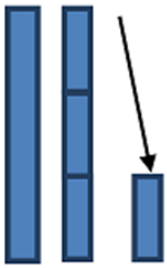
They also need to understand and work with certain principles, i.e. that it is:

- The inverse of multiplication
- Not commutative i.e. 15 ÷3 is not the same as 3 ÷ 15
- Not associative i.e. 30 ÷ (5 ÷ 2) is not the same as (30 ÷ 5) ÷ 2

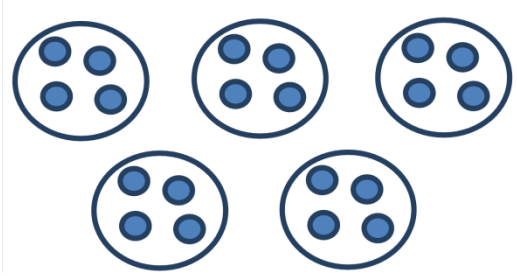
As with multiplication, **scaling** and **grouping** relate to division. In division we also have a third structure which is **sharing**. (See below)

Scaling (ratio, decreasing in equal parts of something).

Children first meet this when exploring halving.



Grouping relates to dividing into equal groups



How many groups of 4 are there in 20?

5 groups of 4 in 20.

20 ÷ 4 = 5

Sharing relates to sharing out a group of objects or quantity

Four children share 12 cookies. How many can they have each?

Division – Early Years Foundation Stage

ELGs

Numerical Patterns:

Pupils should be taught to:

*Verbally count beyond 20, recognising the pattern of the counting system.

*Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as another quantity.

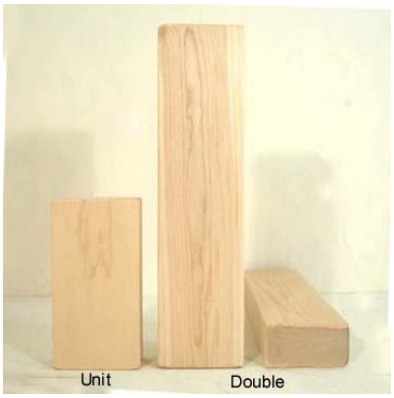
*Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed evenly.

Vocabulary:

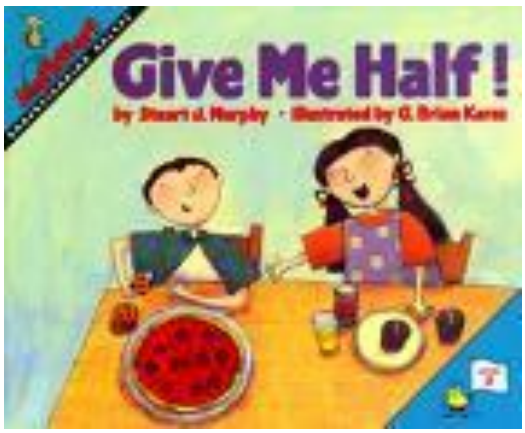
In practical activities and through discussion they will begin to use the vocabulary involved in division; Half of, halve, share, how many, each, divide, group, groups of.

During the EYFS staff introduce the concept of a half of an object or shape through practical activities. For example, when cutting fruit at snack time or play food.

The unit blocks also offer lots of opportunities to explore halves as many of the blocks are made to be half the size of another block.



Children explore sharing sets of objects into equal groups using small world, objects and counters in a variety of contexts.



Division – Year One

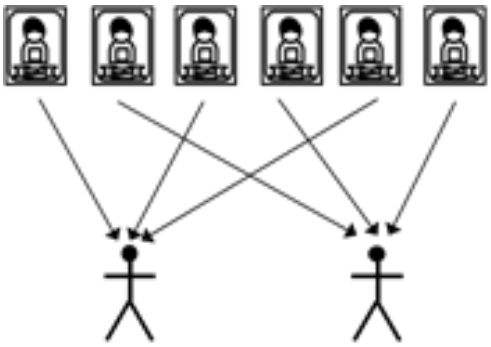
Related NC Statutory Requirements for Year 1:
Pupils should be taught to:
*Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Vocabulary:
Halve, share, share equally, one each, two each, three each…
Group in pairs, threes… tens, equal groups of.

In Year 1, children will continue to solve division problems using practical equipment and jottings.

They should use the equipment to share objects and separate them into groups, answering questions such as 'If we share these six apples between the three of you, how many will you each have? How do you know?' 'If six football stickers are shared between two people, how many do they each get?'

They may solve both of these types of question by using a 'one for you, one for me' strategy until all of the objects have been given out.



Children are introduced to the concept of simple remainders in their calculations at this practical stage, being able to identify that the groups are not equal and should refer to the remainder as '… left over'.

Children are introduced to the concept of simple remainders in their calculations at this practical stage, being able to identify that the groups are not equal and should refer to the remainder as '… left over'.

"Three each and one left-over"

Division – Year Two

Related NC Statutory Requirements for Year 2:

Pupils should be taught to:

*Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.

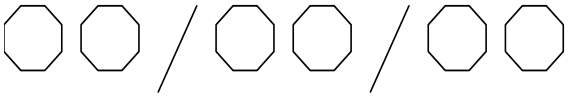
*Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs.

*Show that multiplication of two numbers can be done in any order, commutative, and division of one number by another cannot.

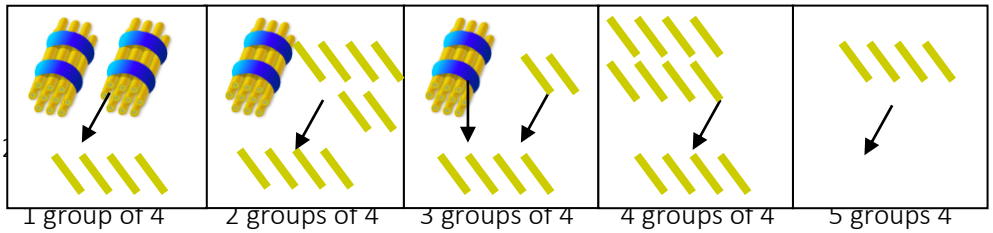
*Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in.

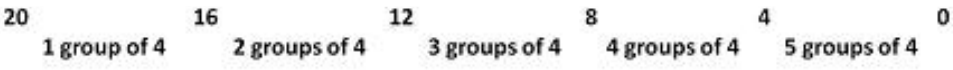
Children will utilise practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation, e.g.

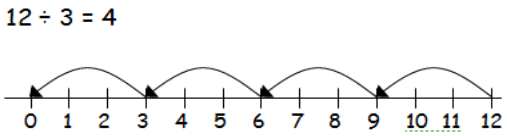
There are 6 sweets, how many people can have two sweets each?

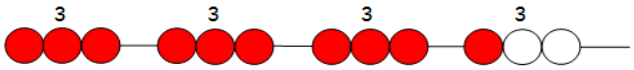


Straws, bead strings and number lines are also good resources for helping children to develop their conceptual understanding of division:

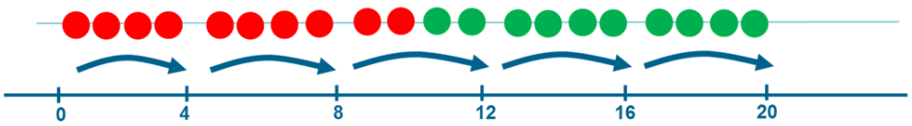








The bead bar will help children with interpreting division calculations such as 10 ÷ 5 as 'how many 5s make 10?'



Many children find grouping by counting on easier than counting back.

Numicon is used as a resource for continuing to develop children's understanding of division with a remainder.

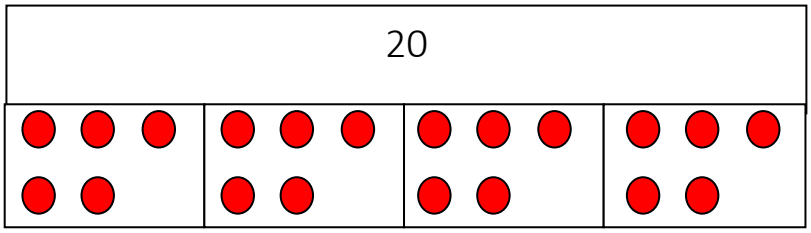
Vocabulary:
Halve, share, share equally one each, two each, three each…
group in pairs, threes… tens equal groups of
÷, divide, divided by, divided into, left, left over, remainder.

Using a 'Bar Model' to solve a division problem:
Children towards the end of Year 2 may be taught to solve a division problem using a bar model.

'Grace's Dad has 20 sweets. He gives Grace and 3 friends an equal amount of sweets. How many sweets will they each receive?'

The children are taught to identify the key information in the problem (as underlined above). They select 20 as the total to be split and recognise the four groups to split the amount between, by identifying Grace and her three friends. The word 'equal' indicates the same number of items must be placed in each group and 'How many sweets' and 'each receive' point towards 'sharing' or 'division'.

The children then set up the following bar model:



Many children will recognise from their work with Numicon and other resources and be able to 'chunk' the 20 into 5s (using number bonds to 10/20 also).

Alternatively, children may count the 20 'sweets' into each group one at a time until there are none left. Then each single 'group' is counted to ensure each of the four children received a 'fair' or 'equal' amount.

Children would then record the answer in a short sentence such as 'each child gets 5 sweets'.

(This method also can be used in a calculation where there is a remainder remainder).

SECTION TWO

Calculation in Key Stage Two

Calculation in Key Stage Two - Prerequisite Skills:

Number and place value:

Pupils should be able to:

- Count in steps of 2, 3, and 5 from 0, and count in tens from any number, forward or backward
- Recognise the place value of each digit in a two-digit number (tens, ones)
- Identify, represent and estimate numbers using different representations, including the number line
- Compare and order numbers from 0 up to 100; use <, > and = signs
- Read and write numbers to at least 100 in numerals and in words; and use place value and number facts to solve problems

Addition:

Pupils should be able to:

- Use concrete objects and pictorial representations, including those involving numbers, quantities and measures
- Applying their increasing knowledge of mental and written methods
- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including
  - a two-digit number and ones
  - a two-digit number and tens
  - two two-digit numbers
  - adding three one-digit numbers
- Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems

Subtraction:

Pupils should be able to:

- Subtract numbers using concrete objects, pictorial representations, and mentally including:
  - a two-digit number and ones
  - a two-digit number and tens
  - two two-digit numbers

Multiplication:

Pupils should be able to:

- Recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×), and equals (=) signs
- Show that multiplication of two numbers can be done in any order (commutative)
- Solve problems involving multiplication using materials, arrays, repeated addition, mental methods and multiplication, including problems in contexts

Division:

Pupils should be able to:

- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- Calculate mathematical statements for multiplication and division within the multiplication; tables and write them using the multiplication (×), division (÷) and equals (=) signs
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts

STAGES IN ADDITION

In developing a written method for addition, it is important that children understand the concept of addition, in that it is:

- Combining two or more groups to give a total or sum
- Increasing an amount

In order to carry out their calculations relating to addition efficiently, they also need to develop an understanding that it is:

- The inverse of subtraction
- Commutative i.e. 5 + 3 = 3 + 5
- Associative i.e. 5 + 3 + 7 = 5 + (3 + 7)

Addition – Year Three

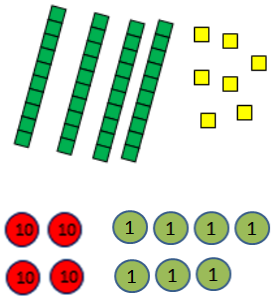
Related NC Statutory Requirements for Year 3:
Pupils should be taught to:
*Add numbers mentally, including:
*A three-digit number and ones.
*A three-digit number and tens.
*A three-digit number and hundreds.
*Add numbers with up to three digits, using formal written methods of columnar addition and subtraction.
*Estimate the answer to a calculation and use inverse operations to check answers.
*Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Vocabulary:
Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange.
See also Y1 and Y2.

Mental Strategies:
Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.
The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged. This will help to develop children's understanding of working mentally.
Children should continue to partition numbers in different ways. They should be encouraged to choose the mental strategies which are most efficient for the numbers involved, e.g.

- Add the nearest multiple of 10, then adjust such as 63 + 29 is the same as 63 + 30 – 1;
- Counting on by partitioning the second number only such as 72 + 31 = 72 + 30 + 1 = 102 + 1 = 103. They can also use a 100 square to help with this showing that the resulting is L shaped (as they count down in tens then across); or
- Mentally partitioning both numbers then adding tens and units before recombining.

Manipulatives can be used to support mental imagery and conceptual understanding. Children need to be shown how these images are related e.g.
What's the same? What's different?



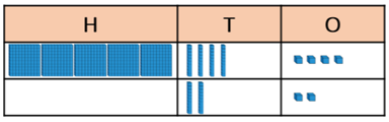
Missing number problems using a range of equations as in Year 1 and 2 but with appropriate, larger numbers.

Partition into tens and ones:
Partition both numbers and recombine.
Count on by partitioning the second number only e.g.
247 + 125 = 247 + 100 + 20+ 5
= 347 + 20 + 5
= 367 + 5
= 372
Children need to be secure adding multiples of 100 and 10 to any three-digit number including those that are not multiples of 10.

Towards a Written Method:

In year 3, children are taught to add 2 and 3 digit numbers using concrete, pictorial and formal written methods of column addition.

Children use concrete equipment such as Base 10 or counters to add two numbers without any need for exchanging.



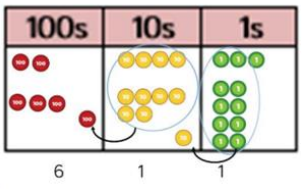
544 + 22 = ?

116 + 343= 459

```
  343
+ 116
  459
```

When initially moving onto column addition, we use numbers that do not cross 10 or 100 and therefore do not require an exchange.

It is important that children realise the digits in the tens column represent 40 and 10 and not 4 and 1.



When beginning to look at the need to exchange numbers, children use place value counters or Base 10 to add HTO + TO, HTO + HTO etc.

When there are 10 ones in the 1s column - we exchange them for 1 ten.
When there are 10 tens in the 10s column - we exchange them for a 1 hundred.

243 + 368 = ?

245 + 84 =

```
  245
+  84
  329
   1
```

Once the method is secure, the children can move on to abstract column addition with exchanging. Children should always start their addition by adding the ones column and ensure any carried numbers are written below the bottom line.

Addition – Year Four

Related NC Statutory Requirements for Year 4:

*Add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate.

*Estimate and use inverse operations to check answers to a calculation.

*Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

Vocabulary:

Add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.

Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.
The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.
Children should continue to partition numbers in different ways.

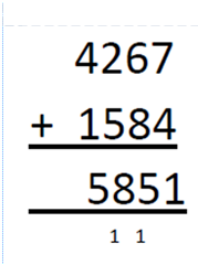
They should be encouraged to choose from a range of strategies:

- Counting forwards and backwards: 124 – 47, count back 40 from 124, then 4 to 80, then 3 to 77
- Reordering: 28 + 75, 75 + 28 (thinking of 28 as 25 + 3)
- Partitioning: counting on or back: 5.6 + 3.7, 5.6 + 3 + 0.7 = 8.6 + 0.7
- Partitioning: bridging through multiples of 10: 6070 – 4987, 4987 + 13 + 1000 + 70
- Partitioning: compensating – 138 + 69, 138 + 70 - 1
- Partitioning: using 'near' doubles - 160 + 170 is double 150, then add 10, then add 20, or double 160 and add 10, or double 170 and subtract 10
- Partitioning: bridging through 60 to calculate a time interval – What was the time 33 minutes before 2.15pm?
- Using known facts and place value to find related facts.

Missing number/digit problems:

Written methods (progressing to 4-digits):

Children are taught to add numbers up to 4 digits consolidating their use of the traditional column method, including carrying ones, tens and hundreds.



Children should already be familiar with the column method from year 3 but it is important to recap the method again, ensuring children understand the value of the digits and that they represent.

It is important children remember and understand that:

The ones must be added first.
Exchanged numbers are written underneath the bottom line.
The place value of the digits is important. It is not 6 add 8, it is 6 tens add 8 tens.

Addition – Year Five

Related NC Statutory Requirements for Year 5:
Pupils should be taught to:
*Add whole numbers with more than 4 digits, including using efficient written methods (columnar addition).
*Add numbers mentally with increasingly large numbers.
*Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.
*Solve addition multi-step problems in contexts, deciding which operations and methods to use and why.

Vocabulary
tens of thousands boundary,
Also see previous years

Generalisation:
Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9.
What do you notice about the differences between consecutive square numbers?
Investigate a – b = (a-1) – (b-1) represented visually.

Some Key Questions:
What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

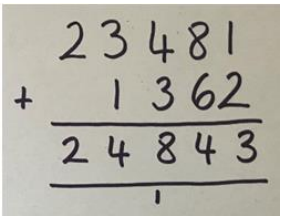
Mental Strategies:
Children should continue to count regularly, on and back, now including steps of powers of 10.
The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.
Children should continue to partition numbers in different ways.

They should be encouraged to choose from a range of strategies:

- Counting forwards and backwards in tenths and hundredths: 1.7 + 0.55
- Reordering: 4.7 + 5.6 – 0.7, 4.7 – 0.7 + 5.6 = 4 + 5.6
- Partitioning: counting on or back - 540 + 280, 540 + 200 + 80
- Partitioning: bridging through multiples of 10:
- Partitioning: compensating: 5.7 + 3.9, 5.7 + 4.0 – 0.1
- Partitioning: using 'near' double: 2.5 + 2.6 is double 2.5 and add 0.1 or double 2.6 and subtract 0.1
- Partitioning: bridging through 60 to calculate a time interval: It is 11.45. How many hours and minutes is it to 15.20?
- Using known facts and place value to find related facts.
- Missing number/digit problems:

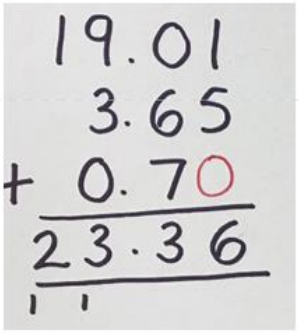
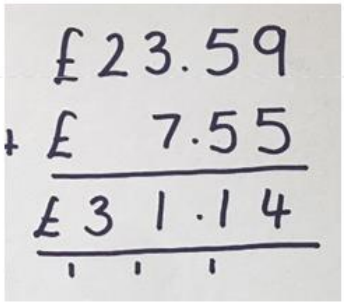
Mental methods:
Should continue to develop, supported by a range of models and images, including the number line. Children should practise with increasingly large numbers to aid fluency
e.g. 12462 + 2300 = 14762

Written methods (progressing to more than 4-digits):
Children use the column method to add decimal numbers in the context of money and measures. It is important that children have place value skills beyond 4 digits here and fully understand what a decimal number represents.



Children start working with numbers greater than 4 digits, including numbers in the ten thousands and hundred thousands.

When using column addition with decimal numbers, such as money, children need to make sure the decimal points are all in line and remembered when writing the answer.
This helps to ensure the correct values are added together.
Children can write in the missing values using a zero as a place holder to help them when calculating the answer, as shown by the red zero in the example below.

	 19.01 3.65 + 0.70 23.36  £23.59 + £ 7.55 £31.14

Addition – Year Six

Related NC Statutory Requirements for Year 6:
Pupils should be taught to:
*Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the efficient written method of long multiplication.
*Perform mental calculations, including with mixed operations and large numbers.
*Identify common factors, common multiples and prime numbers.
*Use their knowledge of the order of operations to carry out calculations involving the four operations.
*Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
*Solve problems involving addition, subtraction, multiplication and division.
*Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

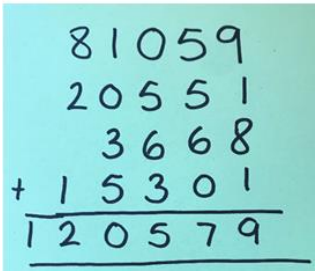
Vocabulary:
See previous years.

Mental Strategies:
Consolidate previous years.

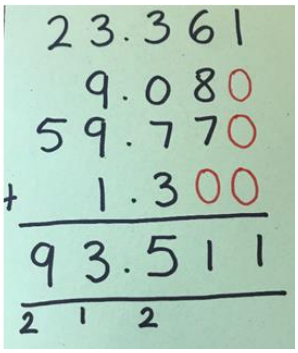
Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. 20 – 5 x 3 = 5; (20 – 5) x 3 = 45

Mental methods:
Should continue to develop, supported by a range of models and images, including the number line.

Written methods:
As Year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured.
Continue calculating with decimals, including those with different numbers of decimal places.

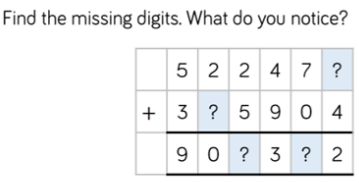


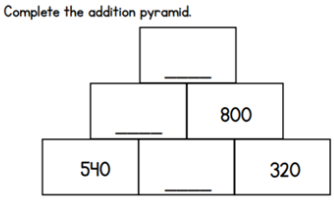
We continue to teach children how to add multiple integers with 4 digits or more using column addition.



Children use their knowledge of the decimal point to line up their digits correctly in the columns. Zero place holders can be added to support place value, as shown by the red zeros in the second example.

Throughout the school, we give children the opportunity to apply their knowledge of addition to different contexts and problems. The examples below relate specifically to Year 6.





	Problem Solving: Teachers should ensure that pupils have the opportunity to apply their knowledge in a variety of contexts and problems (exploring cross curricular links) to deepen their understanding. **Generalisations:** Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering. Sometimes, always or never true? Subtracting numbers makes them smaller. **Some Key Questions:** What do you notice? What's the same? What's different? Can you convince me? How do you know?

Additional Addition Resources

The videos below show a number of the methods we use within school to teach addition strategies.

Using Base 10:

https://www.youtube.com/watch?list=PLjKMtUogeo6VNVG1Aq48VCXIj0iybZnmN&v=f4GEkr3pFzs&feature=emb_logo

Using Counters:

https://www.youtube.com/watch?list=PLjKMtUogeo6VNVG1Aq48VCXIj0iybZnmN&v=324wRW0Guxg&feature=emb_logo

Formal methods for column addition:

https://www.bbc.co.uk/bitesize/topics/zy2mn39/articles/z3kmrwx

https://www.youtube.com/watch?time_continue=92&v=5nex9ScUclU&feature=emb_logo

STAGES IN SUBTRACTION

In developing a written method for subtraction, it is important that children understand the concept of subtraction, in the form of:

- Removal of an amount from a larger group (take away)
- Comparison of two amounts (difference)

They also need to understand and work with certain principles, i.e. that it is:

- The inverse of addition
- Not commutative i.e. 5 - 3 is not the same as 3 - 5
- Not associative i.e. 10 - 3 - 2 is not the same as 10 - (3 - 2)

Subtraction – Year Three

Related NC Statutory Requirements for Year 3:
Pupils should be taught to:
*Subtract numbers mentally, including:
*A three-digit number and ones.
*A three-digit number and tens.
*A three-digit number and hundreds.
*Subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.
*Estimate the answer to a calculation and use inverse operations to check answers.
*Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Vocabulary:
Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange.
See also Y1 and Y2.

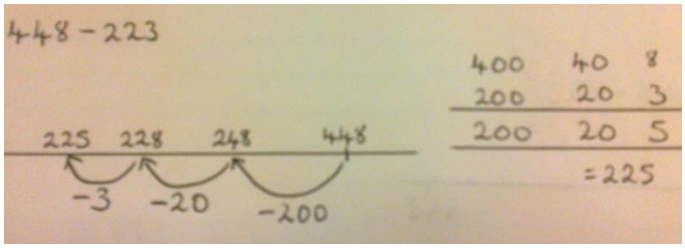
Mental Strategies:
Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.
The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged.
Children should continue to partition numbers in difference ways.
They should be encouraged to choose the mental strategies which are most efficient for the numbers involved, e.g. counting up (difference, or complementary addition) for 201 – 198; counting back (taking away / partition into tens and ones) for 201 – 12.

Generalisations:
Noticing what happens to the digits when you count in tens and hundreds.
Odd – odd = even etc.
Inverses and related facts – develop fluency in finding related addition and subtraction facts.
Develop the knowledge that the inverse relationship can be used as a checking method.

Key Questions:
What do you notice? What patterns can you see?

When comparing two methods alongside each other: What's the same? What's different? Look at this number in the formal method; can you see where it is in the expanded method / on the number line

Missing number problems e.g. □ = 43 – 27; 145 – □ = 138; 274 – 30 = □; 245 – □ = 195; 532 – 200 = □; 364 – 153 = □

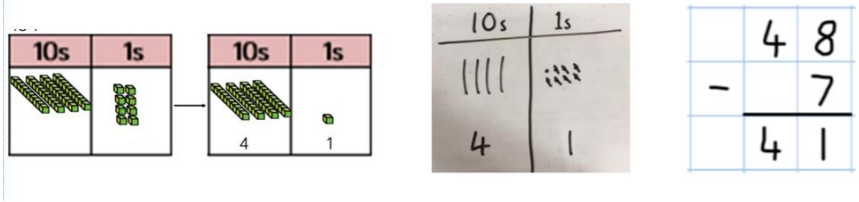


Mental methods:
These should continue to develop, supported by a range of models and images, including the number line.
Children should make choices about whether to use complementary addition or counting back, depending on the numbers involved.

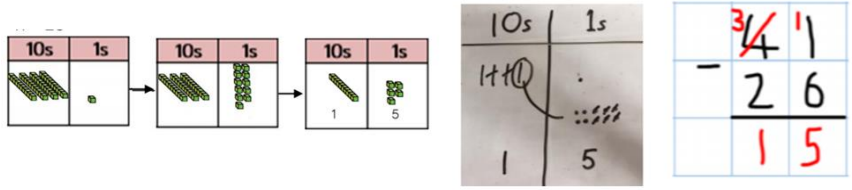
Written methods (progressing to 3-digits):
In Year 3, children are taught to subtract 2- and 3-digit numbers using concrete, pictorial and abstract methods.

Children begin by using Base 10 or counters to represent the subtraction calculation.

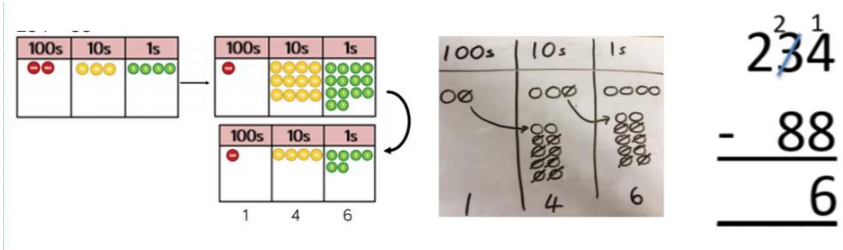
These calculations initially do not require an exchange e.g. 48 - 7 = ?



Children then move onto applying their knowledge to questions that require an exchange e.g 41 - 26 = ?



234 - 88 = ?



Subtraction – Year Four

Related NC Statutory Requirements for Year 4:
Pupils should be taught to:
*Subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate.
*Estimate and use inverse operations to check answers to a calculation.
*Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

Vocabulary:
Add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.

Generalisations:
Investigate when re-ordering works as a strategy for subtraction. E.g. 20 – 3 – 10 = 20 – 10 – 3, but 3 – 20 – 10 would give a different answer.

Some Key Questions:
What do you notice? What's the same? What's different?
Can you convince me? How do you know?

Mental Strategies:
Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.
The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.
Children should continue to partition numbers in different ways.

They should be encouraged to choose from a range of strategies:

- Counting forwards and backwards: 124 – 47, count back 40 from 124, then 4 to 80, then 3 to 77
- Reordering: 28 + 75, 75 + 28 (thinking of 28 as 25 + 3)
- Partitioning: counting on or back: 5.6 + 3.7, 5.6 + 3 + 0.7 = 8.6 + 0.7
- Partitioning: bridging through multiples of 10: 6070 – 4987, 4987 + 13 + 1000 + 70
- Partitioning: compensating – 138 + 69, 138 + 70 - 1
- Partitioning: using 'near' doubles - 160 + 170 is double 150, then add 10, then add 20, or double 160 and add 10, or double 170 and subtract 10
- Partitioning: bridging through 60 to calculate a time interval – What was the time 33 minutes before 2.15pm?
- Using known facts and place value to find related facts.

Missing number/digit problems: 456 + □ = 710;
1□7 + 6□ = 200; 60 + 99 + □ = 340; 200 – 90 – 80 = □; 225 - □ = 150; □ – 25 = 67; 3450 – 1000 = □; □ - 2000 = 900

Mental methods:
These should continue to develop, supported by a range of models and images, including the number line.

Written methods (progressing to 4-digits):
In Year 4, children learn to subtract using compact column subtraction involving numbers up to 4 digits. Children will continue to use concrete and pictorial representations to support their use of the compact method.

Children use concrete equipment such as Base 10 or counters to subtract numbers with exchanging e.g 3454 –1224 = ?

	Th	H	T	O
	3	4	5	4
–	1	2	2	4
	2	2	3	0

Subtraction – Year Five

Related NC Statutory Requirements for Year 5:
Pupils should be taught to:
*Subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction).
*Subtract numbers mentally with increasingly large numbers.
*Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.
*Solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Vocabulary:
Tens of thousands boundary, add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.

Generalisation:
Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9.
What do you notice about the differences between consecutive square numbers?
Investigate a – b = (a-1) – (b-1) represented visually.

Some Key Questions:
What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

Mental Strategies:
Children should continue to count regularly, on and back, now including steps of powers of 10.
The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.
Children should continue to partition numbers in different ways.

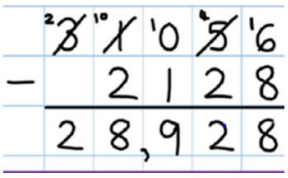
They should be encouraged to choose from a range of strategies:

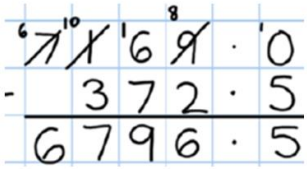
- Counting forwards and backwards in tenths and hundredths: 1.7 + 0.55
- Reordering: 4.7 + 5.6 – 0.7, 4.7 – 0.7 + 5.6 = 4 + 5.6
- Partitioning: counting on or back - 540 + 280, 540 + 200 + 80
- Partitioning: bridging through multiples of 10:
- Partitioning: compensating: 5.7 + 3.9, 5.7 + 4.0 – 0.1
- Partitioning: using 'near' double: 2.5 + 2.6 is double 2.5 and add 0.1 or double 2.6 and subtract 0.1
- Partitioning: bridging through 60 to calculate a time interval: It is 11.45. How many hours and minutes is it to 15.20?
- Using known facts and place value to find related facts.

Missing number/digit problems: 6.45 = 6 + 0.4 + □; 119 - □ = 86; 1 000 000 - □ = 999 000; 600 000 + □ + 1000 = 671 000; 12 462 – 2 300 = □

Mental methods should continue to develop, supported by a range of models and images, including the number line.

Written methods (progressing to more than 4-digits):
Children apply their knowledge from year 3 and 4 to subtract using numbers beyond 4 digits including decimals.





Children will apply their knowledge to calculations and problems, where exchanging will need to take place several times to reach and answer.
Once confident with the larger integers, children will move on to subtracting with decimal numbers in the context of money and measures.
It is important that the decimal numbers in all of the numbers are lined up for accuracy of subtraction.
Where there may be a space in a column, children should put a place holder.
This can be seen in the example opposite where a zero has been placed in the tenths column of the number 7169.

Subtraction – Year Six

Related NC Statutory Requirements for Year 6:
Pupils should be taught to:
*Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.
*Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.
*Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.
*Perform mental calculations, including with mixed operations and large numbers.
*Identify common factors, common multiples and prime numbers.
*Use their knowledge of the order of operations to carry out calculations involving the four operations.
*Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
*Solve problems involving addition, subtraction, multiplication and division.
*Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Vocabulary:
See previous years

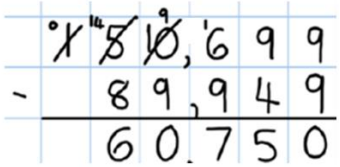
Mental Strategies:
Consolidate previous years.

Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. 20 – 5 x 3 = 5; (20 – 5) x 3 = 45

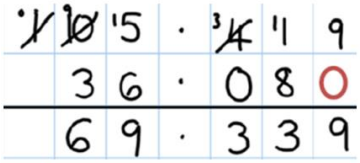
Missing number/digit problems: □ and # each stand for a different number. # = 34. # + # = □ + □ + #. What is the value of □? What if # = 28? What if # = 21
10 000 000 = 9 000 100 + □
7 – 2 x 3 = □; (7 – 2) x 3 = □; (□ - 2) x 3 = 15

Mental methods should continue to develop, supported by a range of models and images, including the number line.

Written methods:
In Year 6, children will learn to subtract with numbers up to 6 digits and beyond including decimals. They will use all previous subtraction skills learned to subtract numbers including those with decimal places. Many of these problems will be in the context of money, measures and worded problems.



Children use the compact method for subtraction to solve problems involving integers up to 6 digits and beyond and solve problems where they will need to use exchanging several times.

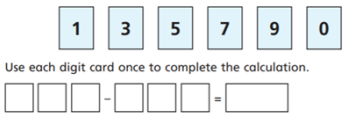


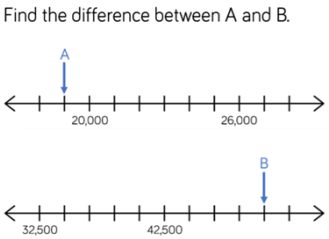
They will also solve problems in context involving increasingly large decimals. They will need to continue to use their knowledge of decimal points to line up their digits and place zeros in any empty columns to support accurate subtraction.

Throughout the school, we give children the opportunity to apply their knowledge of subtraction to different contexts and problems. The examples below relate specifically to year 6.

Generalisations:
Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as BODMAS, or could be encouraged to design their own ways of remembering. Sometimes, always or never true? Subtracting numbers makes them smaller.

Some Key Questions:
What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?





Additional Subtraction Resources

The videos below show a number of the methods we use within school to teach subtraction strategies.

Using Base 10:

https://www.youtube.com/watch?v=Vxq5WCiAN7w&list=PLjKMtUogeo6VNVG1Aq48VCXIj0iybZnmN&index=5

Using Counters:

https://www.youtube.com/watch?v=8Tz_uc3sSCA&list=PLjKMtUogeo6VNVG1Aq48VCXIj0iybZnmN&index=8

https://www.youtube.com/watch?v=YA9p13JcIk0&list=PLjKMtUogeo6VNVG1Aq48VCXIj0iybZnmN&index=7

Formal methods for column subtraction:

https://www.bbc.co.uk/bitesize/topics/zy2mn39/articles/zc78srd

https://www.youtube.com/watch?v=7Z3ngAx1e84

https://www.youtube.com/watch?v=Y6M89-6106I

Please note:

The last video listed uses the word 'borrow'. We do not use the word 'borrow' as it implies that it will be given back. We use the word 'take' instead.

STAGES IN MULTIPLICATION

In developing a written method for multiplication, it is important that children understand the concept of multiplication, in that it can be understood as:

- Repeated addition

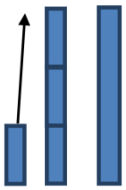
They should also be familiar with the fact that it can be represented as an array

They also need to understand and work with certain principles, i.e. that it is:

- The inverse of division
- Commutative i.e. 5 x 3 is the same as 3 x 5
- Associative i.e. 2 x 3 x 5 is the same as 2 x (3 x 5)

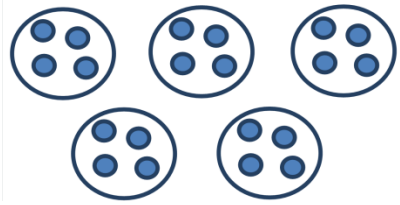
There are two structures for multiplication, these are **scaling** and **grouping**.

Scaling (ratio/ increasing in equal parts of something). This is first met when children explore doubling.



Grouping is repeated addition.

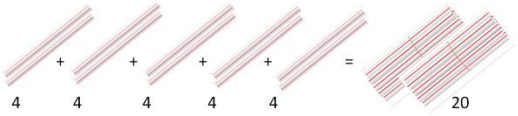
4 + 4 + 4 + 4 + 4 = 20 (4 five times)



5 groups/lots of 4

4 multiplied by 5

4 x 5



Multiplication – Year Three

Related NC Statutory Requirements for Year 3:
Pupils should be taught to:
*Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.
*Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
*Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Vocabulary:
Partition, grid method and Inverse.

Generalisations:
Connecting x2, x4 and x8 through multiplication facts

Comparing times tables with the same times tables which is ten times bigger. If 4 x 3 = 12, then we know 4 x 30 = 120. Use place value counters to demonstrate this.

When they know multiplication facts up to x12, do they know what x13 is? (I.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?)

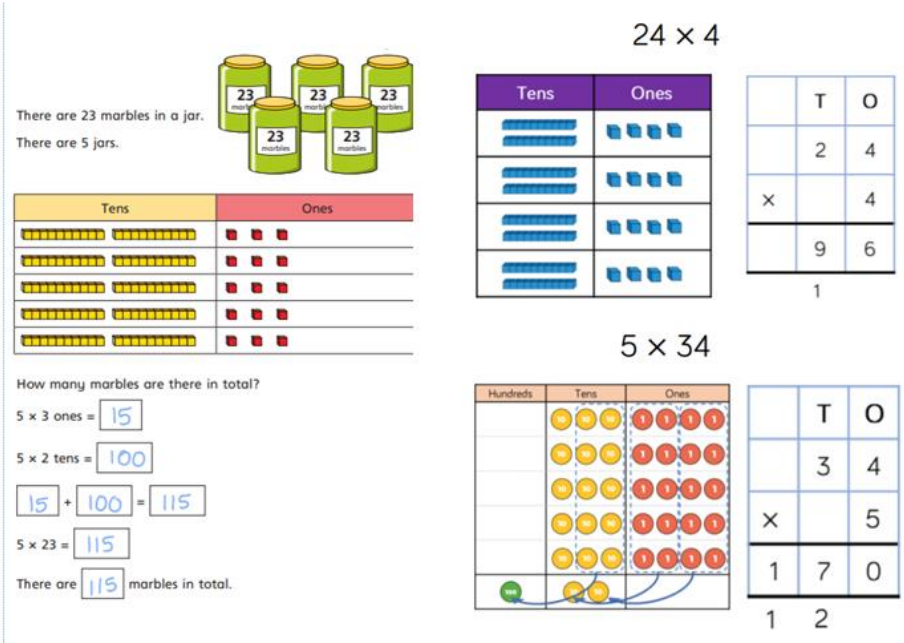
Mental Strategies:
Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.
The number line should continue to be used as an important image to support thinking, and the use of informal jottings and drawings to solve problems should be encouraged.

Children should practise times table facts
3 x 1 =
3 x 2 =
3 x 3 =

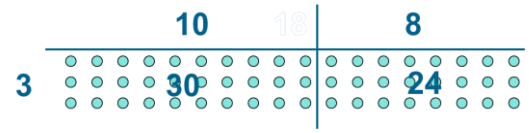
Missing number problems
Continue with a range of equations as in Year 2 but with appropriate numbers.

Mental methods:
Doubling 2 digit numbers using partitioning
Demonstrating multiplication on a number line – jumping in larger groups of amounts
13 x 4 = 10 groups 4 = 3 groups of 4

Written methods (progressing to 2d x 1d):
Developing written methods using understanding of visual images.



NOTE: The grid method is a written method used to teach children multiplication. It involves partitioning numbers into tens and units before they are multiplied and is an efficient way to support children to see the link between this strategy and the standard written methods. It is used within school as an interim step on the way to formal written multiplication methods, however it is not part of our formal calculation policy. It is used at the teacher's discretion, if it is felt necessary, to support those children who may be finding the formal methods tricky.

Some Key Questions: What do you notice? What's the same? What's different? Can you convince me? How do you know?	 37 x 5 = X | 30 | 7 5 | 150 | 35 37 x 5 = 150 + 35 37 x 5 = 185 38 x 62 = X | 60 | 2 30 | 1800 | 60 8 | 480 | 16 38 x 62 = 1800 + 60 + 480 + 16 38 x 62 = 2356 Give children opportunities for children to explore this and deepen understanding using Dienes apparatus and place value counters

Multiplication – Year Four

Related NC Statutory Requirements for Year 4:
Pupils should be taught to:
*Recall multiplication and division facts for multiplication tables up to 12 × 12.
*Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers.
*Recognise and use factor pairs and commutativity in mental calculations.
*Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.
*Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Vocabulary:
Factor.

Generalisations:
Children given the opportunity to investigate numbers multiplied by 1 and 0.

When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?)

Mental Strategies:
Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.
Become fluent and confident to recall all tables to x 12
Use the context of a week and a calendar to support the 7 times table (e.g. how many days in 5 weeks?)
Use of finger strategy for 9 times table.

Multiply 3 numbers together
The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged. They should be encouraged to choose from a range of strategies:

- Partitioning using x10, x20 etc.
- Doubling to solve x2, x4, x8
- Recall of times tables
- Use of commutativity of multiplication

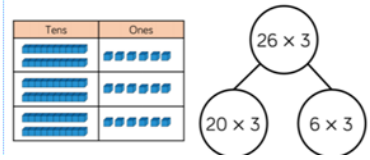
Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits
□2 x 5 = 160

Mental methods:
Counting in multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.
Solving practical problems where children need to scale up. Relate to known number facts. (E.g. how tall would a 25cm sunflower be if it grew 6 times taller?)

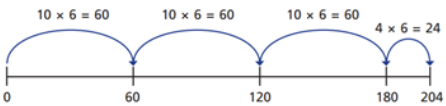
Written methods (progressing to 3d x 2d):

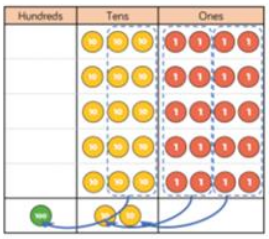
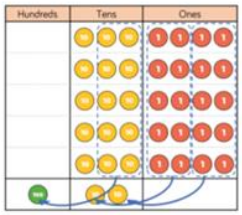
Informal Written Methods:
Children look at informal written methods, such as number lines, part-whole models and place value grids, to explore multiplying 2-digit by 1-digit numbers. They are encouraged to consider when it would be more efficient to use a mental method and when they need to use a written method.

26 x 3 using a part-whole model and a place value grid with Base 10.

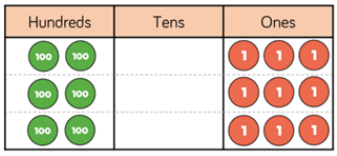


6 x 34 using a number line



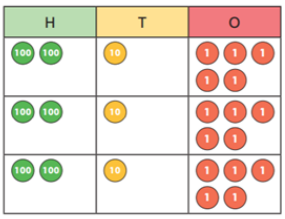
Some Key Questions: What do you notice? What's the same? What's different? Can you convince me? How do you know?	**Formal Written Methods:** Children to embed and deepen their understanding of formal written methods to multiply up 2d x 2d, ensuring these are linked back to their understanding of arrays and place value counters. Children build on their understanding of formal multiplication from year 3 and begin to use short multiplication. The calculations are now more complex than those in year 3 and the children need to consider multiple exchanges between the ones and tens columns and the tens and hundreds columns. Children continue to use visual representations to support the formal method. Many children move on to short multiplication well, however, if children are finding the method tricky, we also teach the expanded method shown in the example below. This is sometimes referred to as the Ladder Method. We then move on to multiplying a 3-digit number by a 1-digit number. Children continue to use visual representations to support the formal multiplication method. We focus particularly on ensuring children know that when there is a 0 in the number, the place value must be kept and not ignored. **Place Value Grid along side the Ladder Method** 5 × 34  H T O 3 4 × 5 2 0 (5 × 4) + 1 5 0 (5 × 30) 1 7 0 **Place Value Grid along side Short Multiplication** 5 × 34  H T O 3 4 × 5 1 7 0 1 2 We then move on to multiplying a 3-digit number by a 1-digit number. Children continue to use visual representations to support the formal multiplication method. We focus particularly on ensuring children know that when there is a 0 in the number, the place value must be kept and not ignored.

3 x 203 = ?



	H	T	O
	2	0	3
×			3
	6	0	9

3 x 215 = ?



	H	T	O
	2	1	5
×			3
	6	4	5
		1	

Once confident with this method, children move away from concrete and pictorial representations and begin to use short multiplication as their first choice of formal method.

	H	T	O
	2	1	7
×			4
	8	6	8
		2	

	H	T	O
	1	0	8
×			6
	6	4	8
		4	

Multiplication – Year Five

Related NC Statutory Requirements for Year 5:

Pupils should be taught to:

*Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.

*Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.

*Establish whether a number up to 100 is prime and recall prime numbers up to 19.

*Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.

*Multiply and divide numbers mentally drawing upon known facts.

*Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

*Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.

*Recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3}).

*Solve problems involving

Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits.

Mental methods:

X by 10, 100, 1000 using moving digits ITP

Use practical resources and jottings to explore equivalent statements (e.g. 4 x 35 = 2 x 2 x 35)

Recall of prime numbers up 19 and identify prime numbers up to 100 (with reasoning)

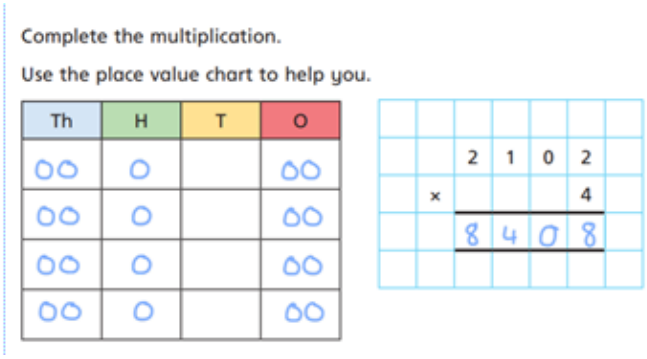
Solving practical problems where children need to scale up. Relate to known number facts.

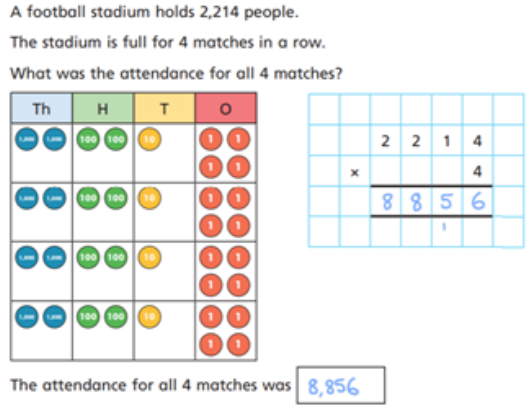
Identify factor pairs for numbers

Written methods (progressing to 4d x 2d):

In Year 5, children build on their understanding of pictorial and formal methods for multiplication and apply this to more complex numbers. The place value grid and counters are used again to support their understanding of the movement and value of the digits within these trickier calculations. They use long multiplication to explore the role of zero when multiplying by a multiple of ten and should be confident when using the method to multiply a 4-digit number by a 2-digit number by the end of Year 5.

Place value grids are used alongside short multiplication when multiplying a 4-digit number by a 1-digit number.





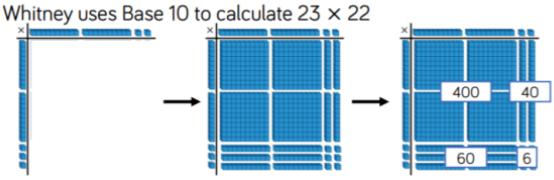
multiplication and division including using their knowledge of factors and multiples, squares and cubes.
*Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.
*Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Vocabulary:
cube numbers, prime numbers, square numbers, common factors, prime number, prime factors and composite numbers.

Generalisation:
Relating arrays to an understanding of square numbers and making cubes to show cube numbers. Understanding that the use of scaling by multiples of 10 can be used to convert between units of measure (e.g. metres to kilometres means to times by 1000)

The Area Model using Base 10

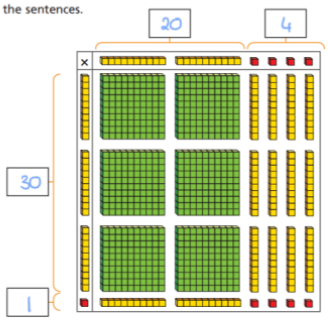
Whitney uses Base 10 to calculate 23 × 22



400 + 40 + 60 + 6= 506

Amir is using base 10 to calculate 31 × 24

a) Add the missing information to the area model and complete the sentences.

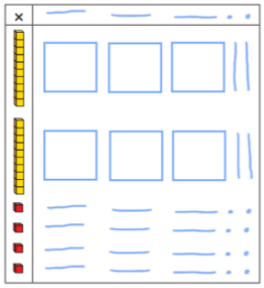


There are 4 ones altogether.
There are 14 tens altogether.
There are 6 hundreds altogether.

The example below is how we would like children to present this method when solving a calculation

24 × 32 = 768

Complete the area model to find the missing number.



Multiplying a 2-digit number by a 2-digit number - formal methods

This is the first time children will be introduced to the role of zero as a place holder when multiplying by a 2-digit number. When multiplying by a multiple of ten, a zero needs to be placed in the ones column before starting the calculation. This can be seen in the example below.

Zero as a place holder

	4	3	
×	1	3	
1	2	9	(43 × 3)
4	3	0	(43 × 10)
5	5	9	

	2	1	
×	1	6	
1	2	6	(21 × 6)
2	1	0	(21 × 10)
3	3	6	

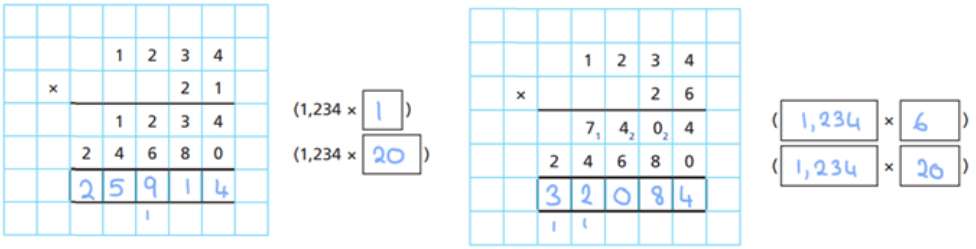
Multiplying a 3- and 4-digit number by a 2-digit number - formal methods

Children use their multiplication knowledge to multiply larger numbers and continue to consolidate their understanding of zero as a place holder.

Multiplying 3 digit numbers by 2 digit numbers:

	2	3	1	
×		1	3	
	6	9	3	(231 × 3)
2	3	1	0	(231 × 10)
3	0	0	3	

	5	1	2	
×		2	4	
2	0	4	8	(512 × 4)
1	0	2	4	0 (512 × 20)
1	2	2	8	8

Some Key Questions: What do you notice? What's the same? What's different? Can you convince me? How do you know? How do you know this is a prime number?	**Multiplying a 4-digit number by a 2-digit number** 

Multiplication – Year Six

Related NC Statutory Requirements for Year 6:
Pupils should be taught to:
*Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.
*Perform mental calculations, including with mixed operations and large numbers.
*Identify common factors, common multiples and prime numbers.
*Use their knowledge of the order of operations to carry out calculations involving the four operations (i.e. BODMAS).
*Solve problems involving multiplication.
*Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Vocabulary:
See previous years but include 'common factor'.

Generalisations:
Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering.
Understanding the use of multiplication to support conversions between units of measurement.

Some Key Questions:
What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

Mental Strategies:
Consolidate previous years.

Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. 20 – 5 x 3 = 5; (20 – 5) x 3 = 45.

They should be encouraged to choose from a range of strategies to solve problems mentally:
- Partitioning using x10, x20 etc.
- Doubling to solve x2, x4, x8
- Recall of times tables
- Use of commutativity of multiplication

If children know the times table facts to 12 x 12. Can they use this to recite other times tables (e.g. the 13 times tables or the 24 times table).

Continue with a range of equations as in Year 2 but with appropriate numbers.

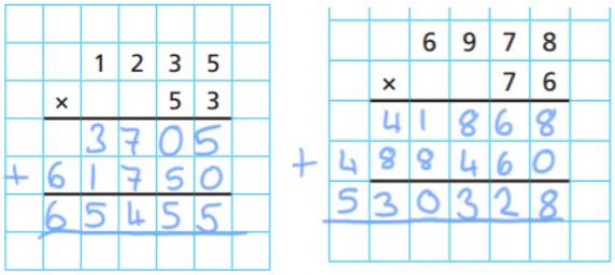
Also include equations with missing digits.

Mental methods:
Identifying common factors and multiples of given numbers
Solving practical problems where children need to scale up. Relate to known number facts.

Written methods:
Continue to refine and deepen understanding of written methods including fluency for using long multiplication.

Multiplying a 4-digit number by a 2-digit number - formal methods

Children consolidate their knowledge of formal multiplication methods, multiplying numbers up to 4 digits.



Additional Multiplication Resources

The videos and websites below show a number of the methods we use within school to teach multiplication as well as explanations of some of the topics covered in this document.

Multiplication using Base 10 (Dienes) with the grid method:
https://www.youtube.com/watch?v=Su2AjrEWvkU

Multiplication using Counters with the grid method:
https://www.youtube.com/watch?v=RRX3AQzYWHM

Short Multiplication:
https://www.youtube.com/watch?v=cBe3RYJRODk

Long Multiplication:
https://www.youtube.com/watch?v=TgwceVtTmtQ

Commutative Law:
https://www.mathsisfun.com/definitions/commutative-law.html

Factors and Common Factors:
https://www.bbc.co.uk/bitesize/topics/z6j2tfr/articles/z72r97h

Prime Numbers:
https://www.bbc.co.uk/bitesize/topics/zfq7hyc/articles/z2q26fr

Square Numbers:
https://www.bbc.co.uk/bitesize/topics/zyhs7p3/articles/z2ndsrd

Cubed Numbers:
https://www.bbc.co.uk/bitesize/topics/zyhs7p3/articles/z2ndsrd

STAGES IN DIVISION

The models for division are the same as those for multiplication which means that they should be relatively easy to teach together.

In developing a written method for division, it is important that children understand the concept of division, in that it is:

- repeated subtraction
- sharing into equal amounts

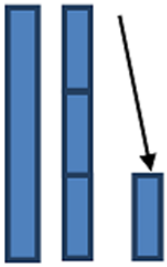
They also need to understand and work with certain principles, i.e. that it is:

- the inverse of multiplication
- not commutative i.e. 15 ÷3 is not the same as 3 ÷ 15
- not associative i.e. 30 ÷ (5 ÷ 2) is not the same as (30 ÷ 5) ÷ 2

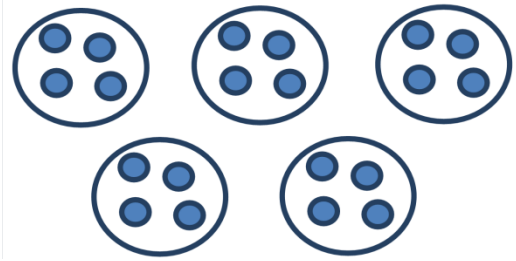
As with multiplication, **scaling** and **grouping** relate to division. In division we also have a third structure which is **sharing**. (See below)

Scaling (ratio, decreasing in equal parts of something).

Children first meet this when exploring halving**.**



Grouping relates to dividing into equal groups



How many groups of 4 are there in 20?
5 groups of 4 in 20
20 divided by 4 = 5
20 ÷ 4 = 5

Sharing relates to sharing out a group of objects or quantity

Four children share 12 cookies. How many can they have each?

Division – Year Three

Related NC Statutory Requirements for Year 3:

Pupils should be taught to:

*Recall and use division facts for the 3, 4 and 8 multiplication tables.

*Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.

*Solve problems, including missing number problems, involving division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

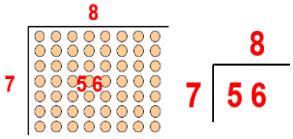
Vocabulary:

See Y1 and Y2, inverse.

Generalisations:

Inverses and related facts – develop fluency in finding related multiplication and division facts.

Develop the knowledge that the inverse relationship can be used as a checking method.



Mental Strategies:

Children should count regularly, on and back, in steps of 3, 4 and 8. Children are encouraged to use what they know about known times table facts to work out other times tables.

This then helps them to make new connections (e.g. through doubling they make connections between the 2, 4 and 8 times tables).

Children will make use multiplication and division facts they know to make links with other facts.

3 x 2 = 6, 6 ÷ 3 = 2, 2 = 6 ÷ 3

30 x 2 = 60, 60 ÷ 3 = 20, 2 = 60 ÷ 30

In lower KS2 we use two structures when talking about division. These are 'sharing equally between' and 'grouping into equal sets.' To understand this fully it is easiest to look at how these are represented:

Sharing

Lucy has 12 sweets and shares them between 3 of her friends. How many sweets does each friend get?

We would say:

12 sweets shared between 3 people gives 4 sweets each.

The answer lies in the value of each equal share.

Grouping

Lucy has 12 sweets. She wants to put them into bags of 3. How many bags will she have?

We would say:

12 sweets put into bags (groups) of 3 gives 4 bags (groups).

The answer lies in the number of equal groups.

Example taken from www.ncetm.org.uk

The grouping structure of division also shows clearly that division is the inverse of multiplication.

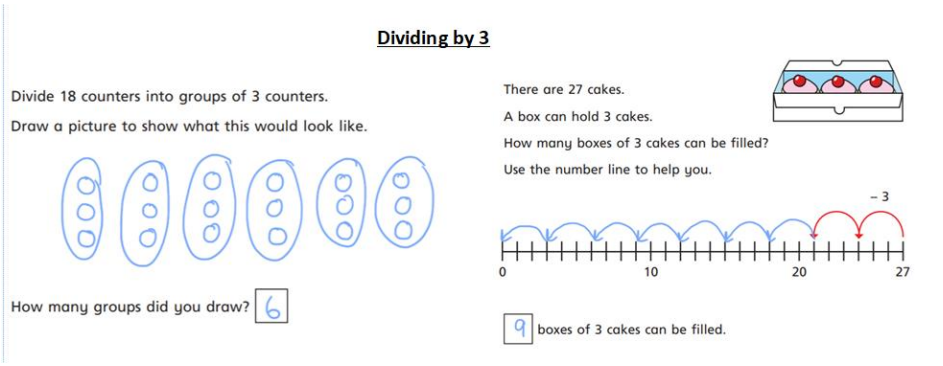
For example, since 4 x 3 = 12, then 12 ÷ 3 must be 4.

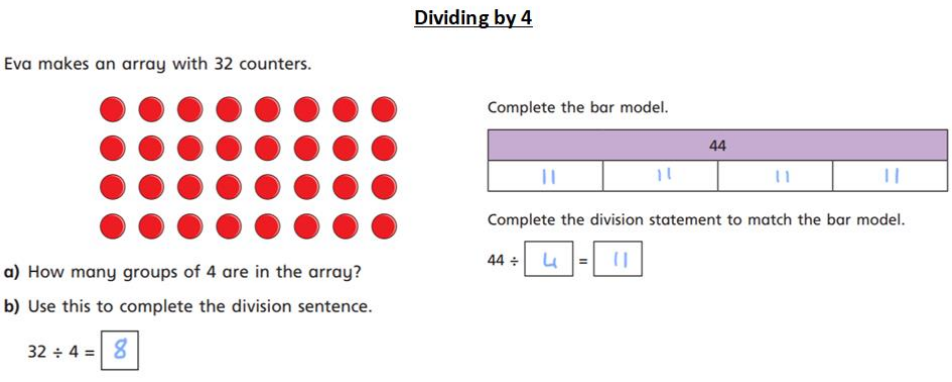
Some Key Questions:
Questions in the context of money and measures that involve remainders (e.g. How many lengths of 10cm can I cut from 81cm of string? You have £54. How many £10 teddies can you buy?)
What is the missing number?

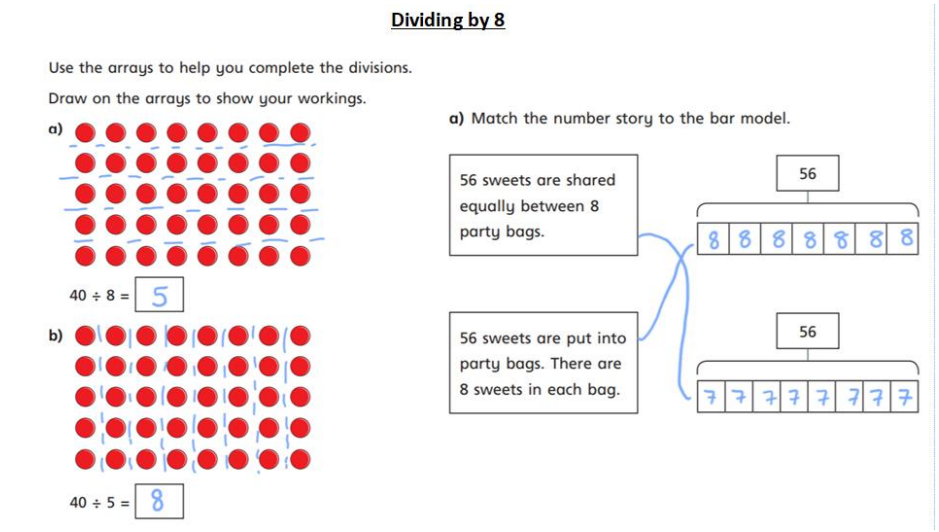
17 = 5 x 3 + __

__ = 2 x 8 + 1

We use concrete and pictorial representations to support the children in understanding division. Below are some of examples of how we introduce division in Year 3.







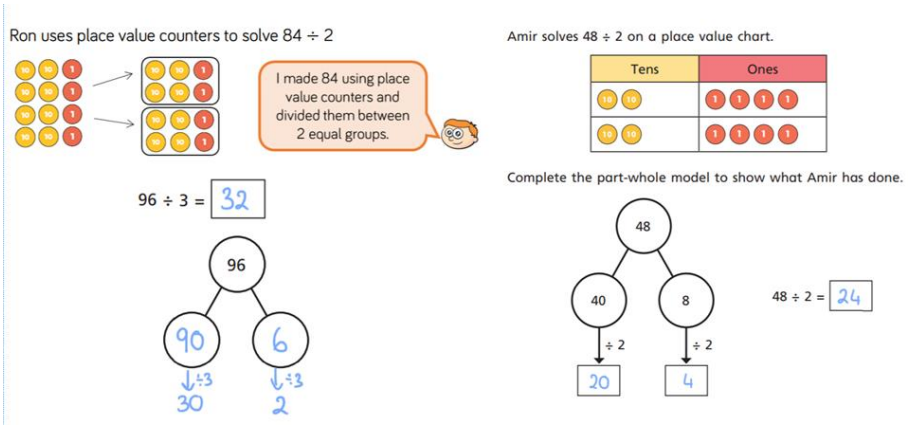
Dividing 2 digit numbers by 1 digit number using concrete and pictorial methods:

We do not introduce formal methods of division until year 4. Children use counters, place value grids, number lines and part whole models to begin their understanding of division beyond their linked division to times tables.

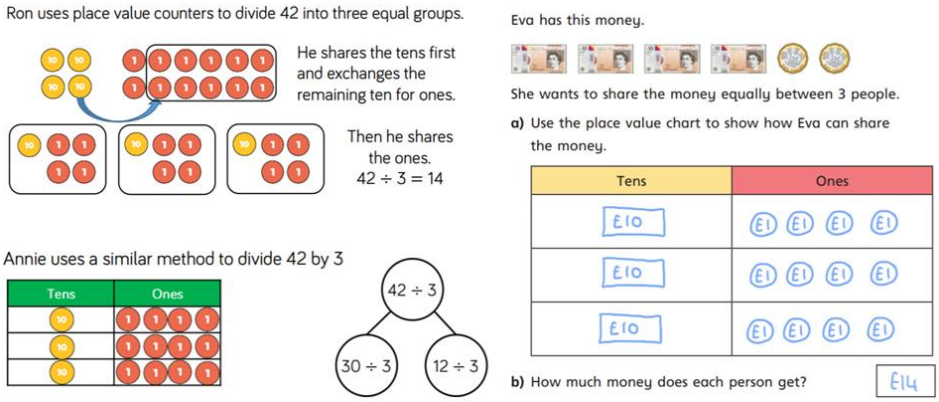
We begin by showing the children how to divide 2 digit numbers by 1 digit numbers using partitioning. Partitioning is when we split a number into smaller units.

For example we can split 457 in to 400, 50 and 7.

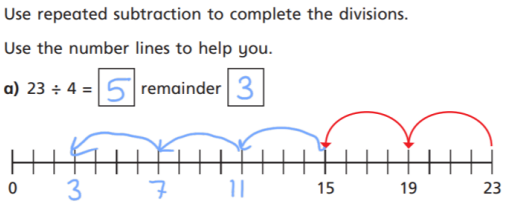
The calculations we use to begin with do not involve an exchange or a remainder.



Once children are familiar with the methods shown above, we move on to calculations that require the children to carry out an exchange, swapping tens for ones.



Finally in Year 3, we build on the previous two steps and introduce remainders using the same methods as previously shown. We also show the children how to use a number line to solve division questions using repeated subtraction.



Division – Year Four

Related NC Statutory Requirements for Year 4:

Pupils should be taught to:

*Recall division facts for multiplication tables up to 12 × 12.

*Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1.

*Recognise and use factor pairs and commutativity in mental calculations.

Vocabulary:

See years 1-3

divide, divided by, divisible by, divided into share between, groups of,

factor, factor pair, multiple

times as (big, long, wide …etc.)

equals, remainder, quotient, divisor inverse.

Towards a formal written method:

Alongside pictorial representations and the use of models and images, children should progress onto short division using a bus stop method.

Place value counters can be used to support children apply their knowledge of grouping. Reference should be made to the value of each digit in the dividend.

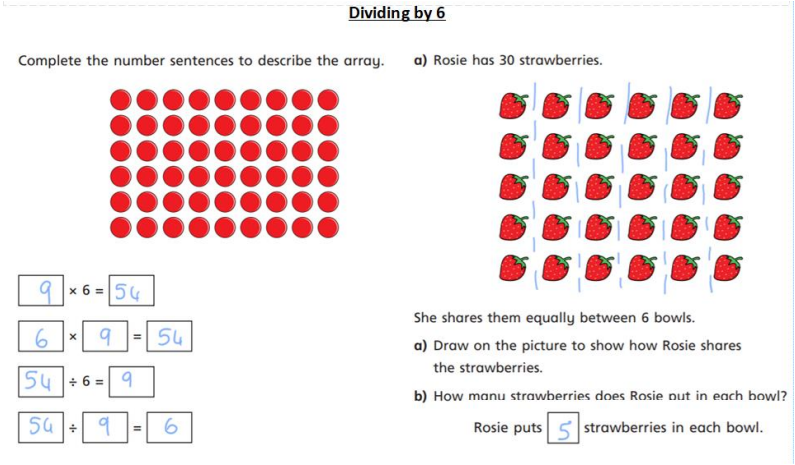
Mental Strategies:

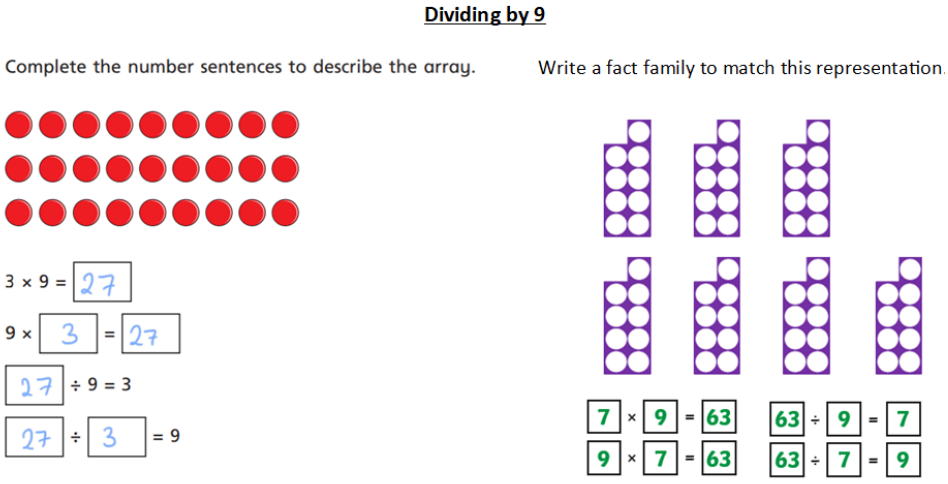
Children should experience regular counting on and back from different numbers in multiples of 6, 7, 9, 25 and 1000.

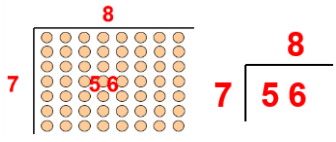
Children should learn the multiplication facts to 12 x 12.

Division linked to times tables:

In Year 4, children focus on learning their 6, 9, 11 and 12 times tables in addition to recapping the tables they have already learned. In contrast to the teaching of division tables in Year 3, Year 4 children learn their division tables together with their associated times tables.

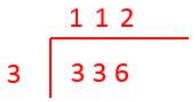


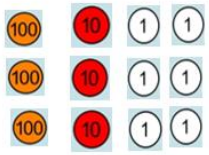




Each digit as a multiple of the divisor:

'How many groups of 3 are there in the hundreds column?'
'How many groups of 3 are there in the tens column?'
'How many groups of 3 are there in the units/ones column?'





When children have conceptual understanding and fluency using the bus stop method without remainders, they can then progress onto 'carrying' their remainder across to the next digit.

Generalisations:

True or false? Dividing by 10 is the same as dividing by 2 and then dividing by 5. Can you find any more rules like this?
Is it sometimes, always or never true that □ ÷ Δ = Δ ÷ □?

Inverses and deriving facts. 'Know one, get lots free!' e.g.: 2 x 3 = 6, so 3 x 2 = 6, 6 ÷ 2 = 3, 60 ÷ 20 = 3, 600 ÷ 3 = 200 etc.

Dividing by 7

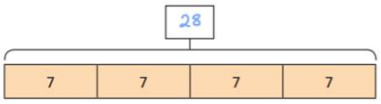
Complete the number sentences to describe the array.

2 × 7 = 14
7 × 2 = 14
14 ÷ 7 = 2
14 ÷ 2 = 7

There are 7 players in a netball team.

a) How many players are there in 4 netball teams?

Label the whole on the bar model

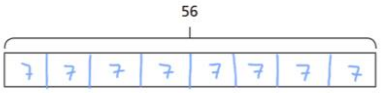


Complete the sentences.

4 × 7 = 28

There are 28 players in 4 netball teams.

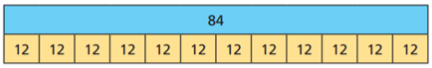
b) If there are 56 players, how many full teams are there



There are 8 full teams.

Dividing by 11 and 12

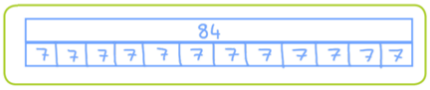
Ron uses a bar model to represent 84 divided by 12



a) Explain Ron's mistake.

He has split his bar into 12 sections and wrote 12 in each.

b) Draw the correct bar model diagram to represent 84 divided by 12



Mr Scott is organising a cricket tournament.

a) There are 11 players in a cricket team.

5 teams have signed up for the tournament.

How many players have signed up?

55

b) Mr Scott needs 132 players signed up to go ahead with the tournament.

How many more teams are needed?

7 more teams are needed.

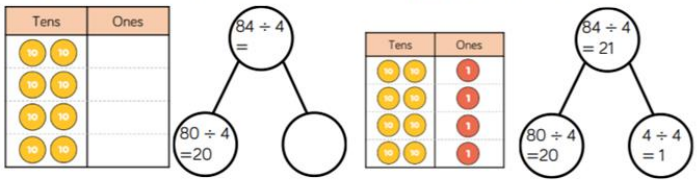
Division of 2 digit numbers by 1 digit numbers:

Children build on their knowledge from year 3 and continue to use concrete and pictorial representations to divide 2 digit numbers by 1 digit numbers. They begin by dividing numbers that are divisible exactly by the divisor and then move onto calculations that require them to exchange between the tens and ones columns.

Dividing a 2 digit number by a 1 digit number - no remainders or exchanging

Jack is dividing 84 by 4 using place value counters.

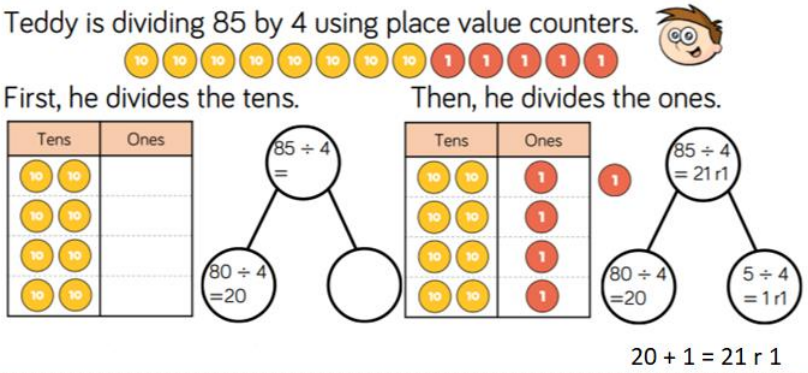
First, he divides the tens. Then, he divides the ones.



20 + 1 = 21

Sometimes, always, never true questions about multiples and divisibility. (When looking at the examples on this page, remember that they **may not** be 'always true'!) E.g.:
*Multiples of 5 end in 0 or 5.
*The digital root of a multiple of 3 will be 3, 6 or 9.
*The sum of 4 even numbers is divisible by 4.

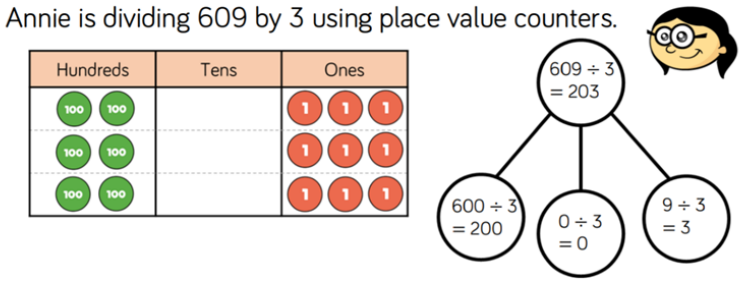
Dividing a 2 digit number by a 1 digit number with remainders and exchanging



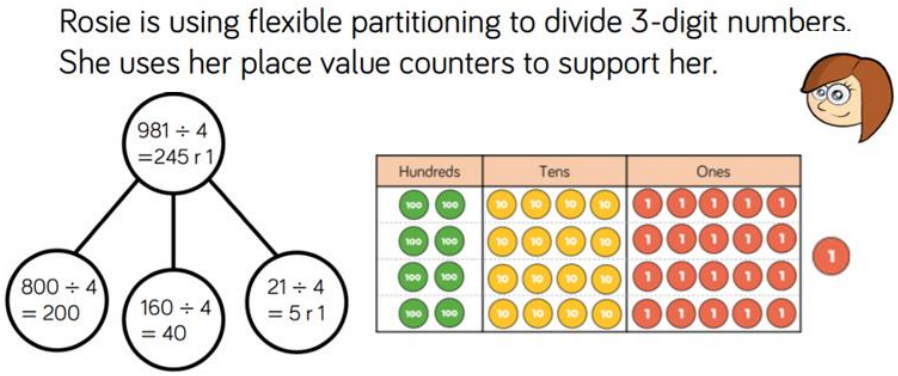
20 + 1 = 21 r 1

Dividing a 2 digit number by a 1 digit number with remainders and exchanging

Once children are secure in dividing 2 digit numbers by 1 digit numbers, we then introduce 3 digit numbers to the calculations.



200 + 0 + 3 = 203



200 + 40 + 5 = 245 r 1

÷ = signs and missing numbers:
Continue using a range of equations as in year 3 but with appropriate numbers.

Sharing, Grouping and using a number line:

Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding. Children should progress in their use of written division calculations:

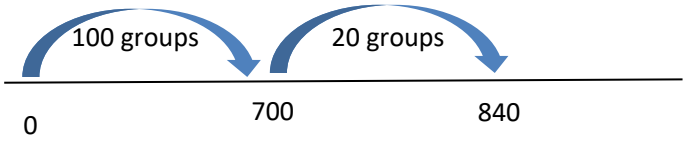
- Using tables facts with which they are fluent.
- Experiencing a logical progression in the numbers they use, for example:
  1. Dividend just over 10x the divisor, e.g. 84 ÷ 7
  2. Dividend just over 10x the divisor when the divisor is a teen number, e.g. 173 ÷ 15 (learning sensible strategies for calculations such as 102 ÷ 17)
  3. Dividend over 100x the divisor, e.g. 840 ÷ 7
  4. Dividend over 20x the divisor, e.g. 168 ÷ 7

All of the above stages should include calculations with remainders as well as without.

Remainders should be interpreted according to the context. (i.e. rounded up or down to relate to the answer to the problem)

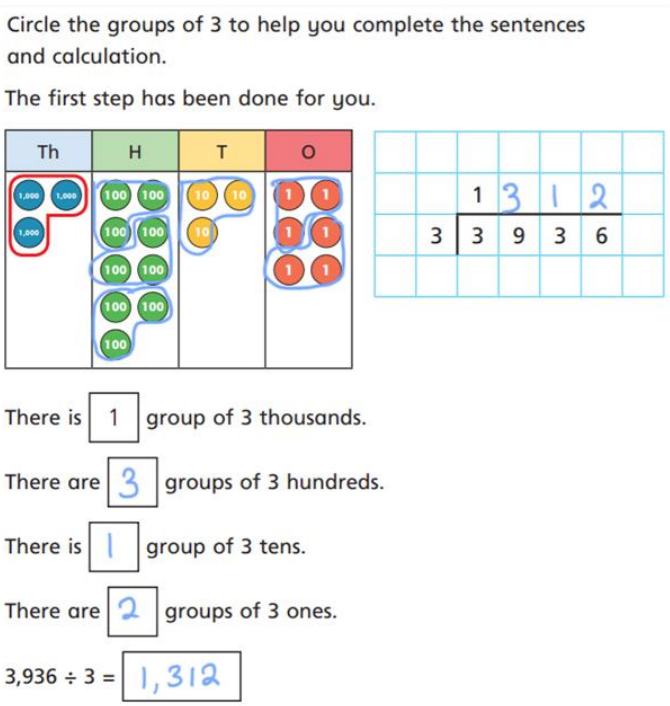
e.g. 840 ÷ 7 = 120

Jottings
7 x 100 = 700
7 x 10 = 70
7 x 20 = 140

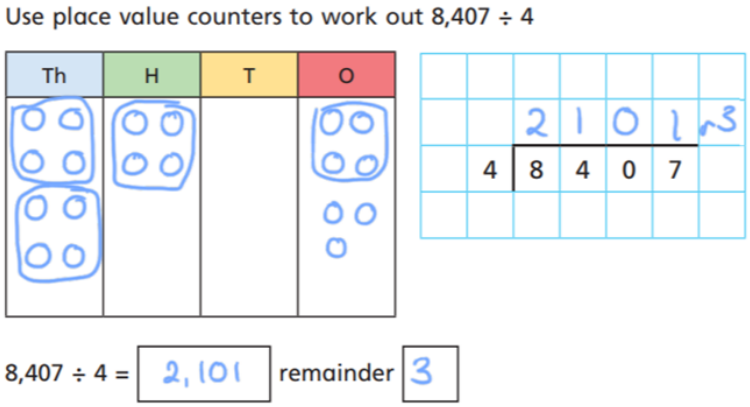


Short Division - Bus Stop Method:

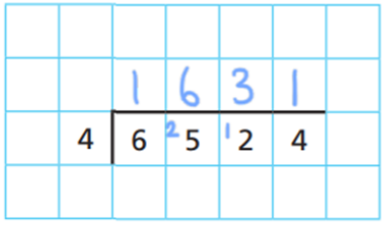
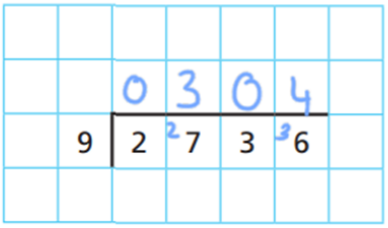
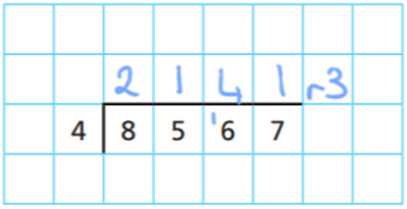
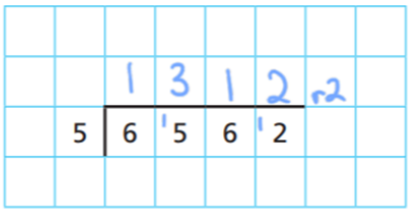
Short division is introduced in Year 4 alongside the use of place value grids and counters. This helps support the children to see how the digits are grouped when being divided and how any carried numbers impact the calculation. As the children move into Year 5, the children work less with visual representations and deal with the formal method for short division more frequently, applying the method to larger and more complex division questions and problems.



Short division with remainders



Children continue to use visual and concrete representations in conjunction with the formal methods until they feel confident to use the formal method alone. This is mainly seen in Year 5.

	 1631 4 6 25 12 4  0304 9 2 27 3 36  2141 r3 4 8 5 16 7  1312 r2 5 6 15 6 12

Division – Year Five

Related NC Statutory Requirements for Year 5:

Pupils should be taught to:

*Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.

*Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.

*Establish whether a number up to 100 is prime and recall prime numbers up to 19.

*Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.

*Multiply and divide numbers mentally drawing upon known facts.

*Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

*Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.

*Recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3}).

*Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes.

*Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.

*Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Mental Strategies:

Children should count regularly using a range of multiples, and powers of 10, 100 and 1000, building fluency.

Children should practice and apply the multiplication facts to 12 x 12.

÷ = signs and missing numbers:

Continue using a range of equations as in year 3 but with appropriate numbers.

Sharing, Grouping and Using a number line:

Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding.

Children should progress in their use of written division calculations:

- Using tables facts with which they are fluent
- Experiencing a logical progression in the numbers they use, for example:
    1. Dividend just over 10x the divisor, e.g. 84 ÷ 7
    2. Dividend just over 10x the divisor when the divisor is a teen number, e.g. 173 ÷ 15 (learning sensible strategies for calculations such as 102 ÷ 17)
    3. Dividend over 100x the divisor, e.g. 840 ÷ 7
    4. Dividend over 20x the divisor, e.g. 168 ÷ 7

All of the above stages should include calculations with remainders as well as without.

Remainders should be interpreted according to the context.
(I.e. rounded up or down to relate to the answer to the problem)

e.g. 840 ÷ 7 = 120

Jottings
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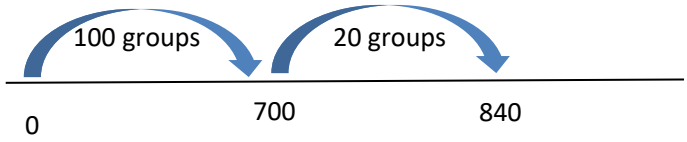
7 x 20 = 140



Formal Written Methods:

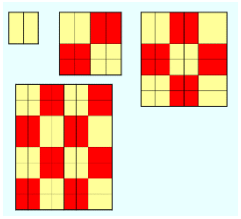
Continued as shown in Year 4, leading to the efficient use of a formal method. The language of grouping to be used (see link from fig. 1 in Year 4)

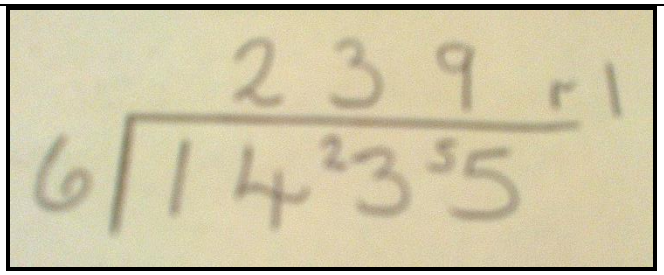
E.g. 1435 ÷ 6

Vocabulary:
See year 4,
common factors,
prime number, prime factors,
composite numbers,
short division,
square number,
cube number,
inverse,
power of.

Generalisations:
The = sign means equality. Take it in turn to change one side of this equation, using multiplication and division, e.g.
Start: **24 = 24**
Player 1: **4 x 6 = 24**
Player 2: **4 x 6 = 12 x 2**
Player 1: **48 ÷ 2 = 12 x 2**

Sometimes, always, never true questions about multiples and divisibility. E.g.:
*If the last two digits of a number are divisible by 4, the number will be divisible by 4.
*If the digital root of a number is 9, the number will be divisible by 9.
*When you square an even number the result will be divisible by 4 (one example of 'proof' shown below).



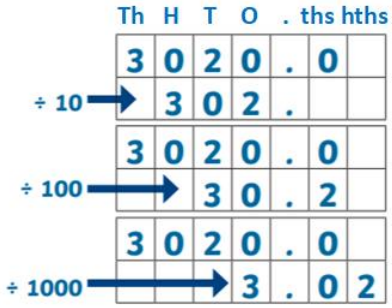


Children begin to practically develop their understanding of how express the remainder as a decimal or a fraction. Ensure practical understanding allows children to work through this (e.g. what could I do with this remaining 1? How could I share this between 6 as well?).

Dividing by 10, 100 and 1000:

Year 5 children build on their use of the place value grid for dividing by 10 and 100 and use it to divide numbers by 1000.
Dividing by 10 - all the digits move 1 place to the right in the place value grid.
Dividing by 100 - all the digits move 2 places to the right in the place value grid.
Dividing by 1000 - all the digits move 3 places to the right in the place value grid.

3020 ÷ 10 = 302
3020 ÷ 100 = 30.2
3020 ÷ 1000 = 3.02



Division – Year Six

Related NC Statutory Requirements for Year 6:

Pupils should be taught to:

*Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.

*Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.

*Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.

*Perform mental calculations, including with mixed operations and large numbers.

*Identify common factors, common multiples and prime numbers.

*Use their knowledge of the order of operations to carry out calculations involving the four operations.

*Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

*Solve problems involving addition, subtraction, multiplication and division.

*Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

÷ = signs and missing numbers:

Continue using a range of equations but with appropriate numbers

Sharing and Grouping and using a number line

Children will continue to explore division as sharing and grouping, and to represent calculations on a number line as appropriate.

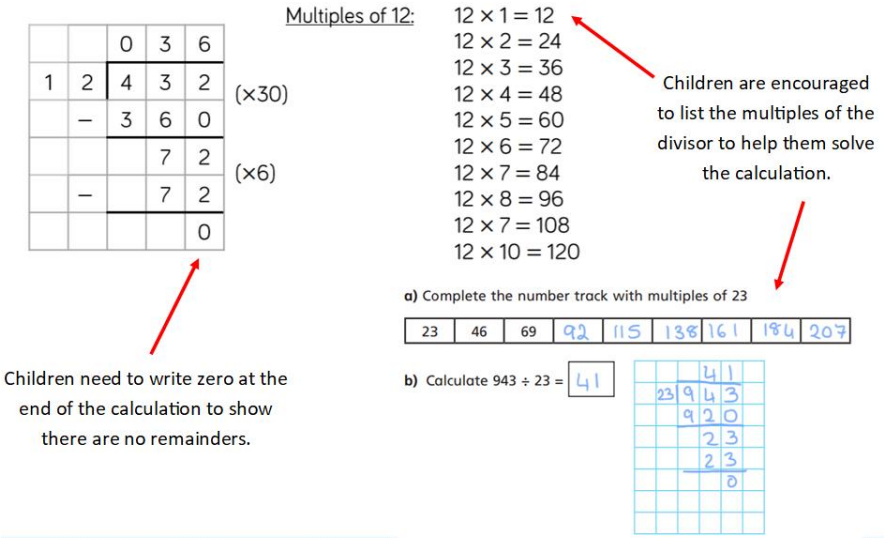
Quotients should be expressed as decimals and fractions

Formal Written Methods – long and short division.

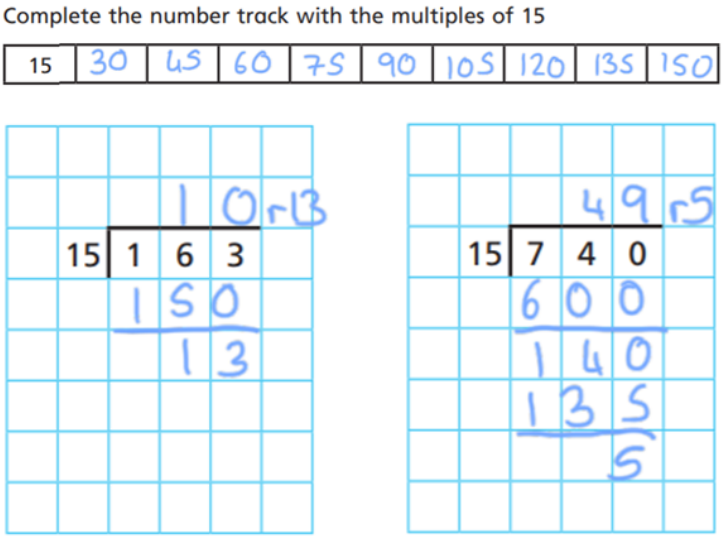
Long Division:

Children in year 6 recap the method for short division (Bus Stop Method) used in years 4 and 5 and move onto exploring long division.

They begin by dividing 3 digit numbers that will divide exactly by the divisor without leaving a remainder.



The next step in teaching long division, is to introduce calculations that involve numbers up to 4 digits as well as those that have a remainder. Children continue to list the multiples of the divisor to help them with their calculations.



Tommy uses this method to calculate 372 divided by 15
He has used his knowledge of multiples to help.

			2	4	r	1	2
1	5	3	7	2			
	–	3	0	0			
			7	2			
	–		6	0			
			1	2			

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

$432 \div 15$ becomes

$$
\begin{array}{r|l}
15 & 432.0 \\
 & 28.8 \\
\hline
 & 30\downarrow \\
 & 132 \\
 & 120 \\
\hline
 & 120 \\
 & 120 \\
\hline
 & 0
\end{array}
$$

The remainders are found at the end of the calculation when they remaining number being dealt with is smaller than the divisor.

Additional Division Resources

The videos and websites below show a number of the methods we use within school to teach multiplication as well as explanations of some of the topics covered in this document.

Grouping and Sharing:

https://www.youtube.com/watch?v=bdglIPNNhuI

Division using concrete resources - straws and counters:

https://www.youtube.com/watch?time_continue=138&v=ysjYJ1Idfb4&feature=emb_logo

https://www.youtube.com/watch?time_continue=165&v=4EcMON3F1yE&feature=emb_logo

Short Division:

https://www.youtube.com/watch?time_continue=339&v=06KXeyhek9c&feature=emb_logo

https://www.bbc.co.uk/bitesize/topics/z36tyrd/articles/zgxdfcw

https://www.theschoolrun.com/what-is-the-bus-stop-method-for-division

Long division:

https://www.theschoolrun.com/what-is-long-division

https://www.youtube.com/watch?v=mZZe9-qD12Q

https://thirdspacelearning.com/blog/teach-long-division-method-ks2-steps/